SYNTONALITY: A NEW SYSTEM OF HARMONY

DANA DIMITRI RICHARDSON

Contact: richardson_dana@hotmail.com

Examples, Figures and Tables

Syntonality

Syntonality is a system of composition designed to provide composers with the ability to control the tension/relaxation gradient at will, produce a coherent flow of harmony, and create a satisfying climax/denouement structure, in short, do everything that was possible to tonal composers, while employing all the chromatic resources of post-tonal composition. It can be defined as a special case of bitonality, limited by the following conditions: 1) there are two and only two simultaneously unfolding keys. 2) These keys are tritone or semitone related so that all 12 tones are accounted for, with two shared between them. 3) The keys are horizontally and vertically fused with the result that they are at first apparent neither to the eye nor to the ear.

Registral fusion creates a new surface in which the identity of the constituent keys dissolves at the level of conscious awareness. The listener is aware only of the succession of integral syntonal harmonies whose properties seemingly bear no relation to their tonal ingredients. However, as will be demonstrated, there is actually an intimate and definable relationship between the two. Thus syntonality posits a deep structure, the simultaneity of two identifiable major keys or **keystreams,** and a surface structure, the integrally heard fused harmonies or **superchords**, their organization into **superkeys**, and the relations between superkeys. The analysis of the surface structure reveals the deep structure whereas the synthesis of the deep structure yields the surface. The two levels of structure can be elegantly defined in terms of the other.

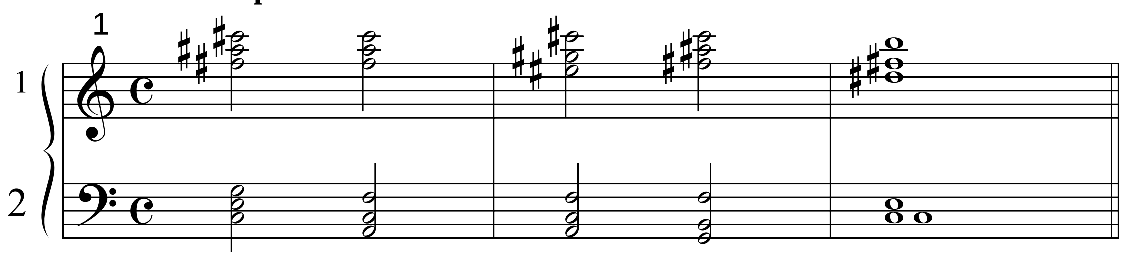
The syntonal harmony is not called forth by the fiat of the composer. It is a referent that is antecedent to any syntonal composition in the way that the triad is given to any tonal composition. It will be come evident that, although the keystreams dissolve in the new surface, the syntonal capacity for harmonic continuity derives its existence from the fused continuity of their constituent seventh-chords. As will be shown, there are eighty-one eight- pc superchords formed by the fusion of two seventh-chords. Every harmony in every syntonal composition must be one of those eighty-one superchords or its subset. A simultaneity that is not one of those harmonies indicates a dissonance that must be resolved. As in tonality, the relatively small number of possible harmonies is limited by principles inherent in the system.

Example 1 is a short bitonal excerpt with very widely separated registers in the keys of C and F#. Both keystreams are immediately perceptible as separate entities, yet a third factor has been added, the sonority of the simultaneous keystreams. [Example 1. Bitonal stratification]

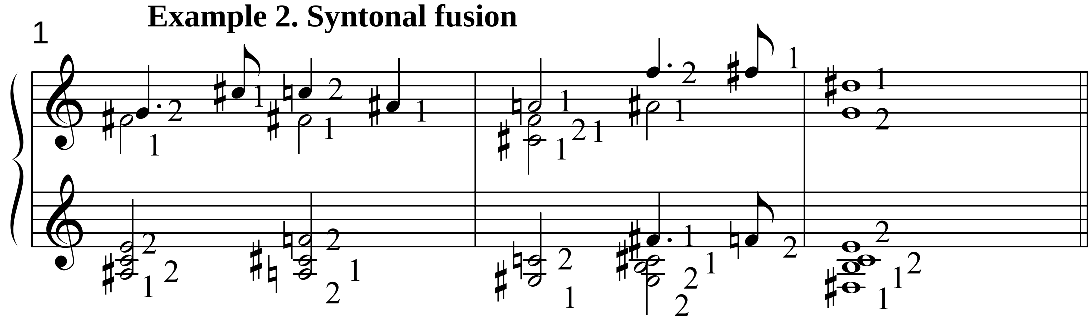
In Example 2, the bitonally stratified harmonies of Example 1 have been registerally fused. Whereas C and F# major are heard as integral, simultaneous tonalities in Example 1, in Example 2, they cannot be heard independently. In fact, the fused simultaneities of Example 2 sound very different from the corresponding stratified simultaneities of Example 1, even though they are composed of the same pcs. They have been dissolved in the syntonal surface through registral fusion. [Example 2. Syntonal fusion]

Now one is a position to appreciate why bitonally has been rejected by many as an analytical tool. The tendency to fuse bitonal keystreams is very strong even when they are mildly registrally stratified. So, in the vast majority of bitonal pieces one does not hear a clearly divided simultaneity; one hears a more or less unified sonority that becomes completely unified under the condition of syntonal fusion. Only when the

Example 1. Bitonal stratification



Example 2. Syntonal fusion



registers are very widely separated, and when the tonal harmonies are sufficiently complete to resist assimilation into the opposing keystream are the simultaneous tonal structures detectable. This is what occurs in Example 1.

Although the integral, simultaneous tonalities of Example 1 are dissolved in the syntonal surface of Example 2 through registral fusion, *the resultant harmonies are the same* (if one assumes the principle of octave equivalence). The numbers to the right of the pitches in Example 2 refer to the keystream in Example 1 from which they were taken. Example 2 takes the bitonal harmonies of Example 1 and redistributes the pitch classes but leaves the pc content of each harmony unchanged. Therefore, the tonal principles that control Example 1 must also control Example 2. This justifies the apparatus of keystream analysis to which I now turn.

Deep Structure: The Keystreams

Constant Vauclain established the foundation of syntonal deep structure, the keystreams, (although he didn't call them that) in two articles, "An Experiment in Musical Texture" and "Bartok: Beyond Bi-modality."[1] The presentation of keystream theory here preserves Vauclain's basic conception but modifies it in a way that I consider necessary to maintain the integrity of the system.

Registral fusion results in new surface with properties not present in either keystream. The keystream analysis extracts the tonal streams, each of which appears on one of two parallel pairs of braced staves. The music in each braced pair must proceed in logical tonal progressions with all dissonances resolving by step. Performance of the two keystreams simultaneously recovers the whole piece. Example 3, a twelve-measure segment from my *Hymn to Pan/IV*, and Example 4, which extracts the keystreams, illustrate the process. [Example 3. Richardson, *Hymn to Pan/IV*, mm. 106-17]

In Example 4, the upper brace of staves begins with the keystream in G major that modulates to C and so on. The lower brace begins with the keystream in Ab major. Changes of key within keystreams are indicated by the capital letters in brackets found between the staves. The tonal harmonies within the keystreams are indicated by Roman numerals with their figures in the standard manner. The dotted lines and slurs indicate dissonance and its resolution. [Example 4. Richardson, *Hymn to Pan/IV*, keystream analysis]

The keystream analysis includes both harmonic and voice leading elements. Purely harmonic considerations will be discussed first.

1) When keystream 1 modulates to C, keystream 2 must modulate. If it did not the two keys, C and Ab, would be related by third and thus fail to fulfill the condition that the keys be either tritonally or semitonally related. In this case, keystream 2 modulates to Db, the most closely related key. Hypothetically, keystream could have modulated to either Gb or B major and still have fulfilled the **tritone/semitone condition**.

2) When carrying out a keystream analysis, the analyst should divide the keystreams so that each follows the path of least resistance. For instance, in Example 4, the modulations should proceed as shown G→C and Ab→Db, not G→Db and Ab→C. Keystream extraction should produce ‘natural’ results. Bizarre and awkward progressions within each keystream are an indication that there is something wrong with the analysis or that the music is not syntonal.

Example 3. Richardson, Hymn to Pan/IV

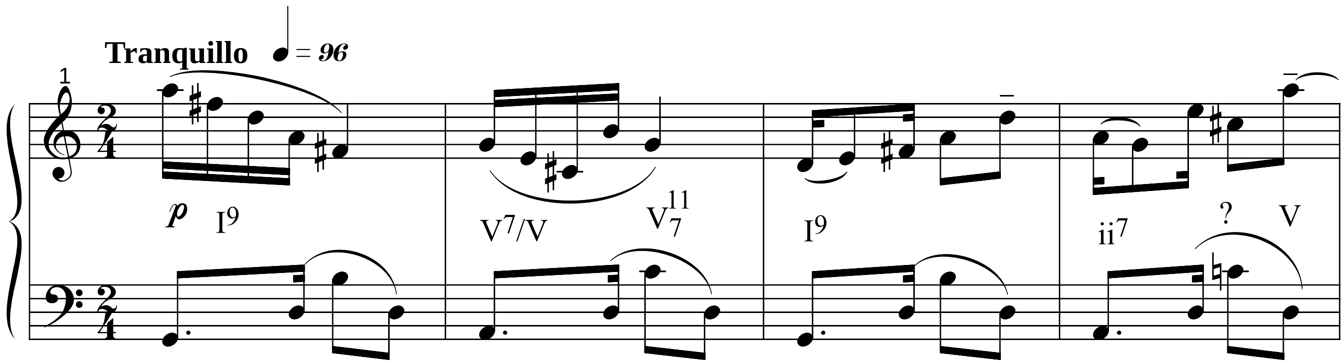
Example 4. Hymn to Pan/IV, keystream analysis

3) Each successive tonality in a keystream must be completely diatonic. There are no secondary dominants, chromatic embellishments or altered tones. Every dominant implies a new key even if it is immediately succeeded by another dominant implying a different key as in a dominant circle of fifths progression. These constraints are necessary to insure that at every instant the keystreams are tritonally or semitonally related.

At this point one may ask "Why is it so important that the keystreams be limited to the relation of a tritone or semitone". There are three reasons:1) To avoid lapsing into monotonality. Imagine a case in which both keystreams were in the same key. If the composer wanted to differentiate his harmonic style from traditional tonal practices, the best he could hope to achieve would be pan-diatonicism or 'white note' music. He certainly would not require an analytical apparatus that divided the texture into keystreams! All the pitches would be heard as referable to a single root with dissonances that might or might not resolve. Now suppose that the keystreams were related by perfect fifth, say G and D major. Example 5, taken from Darius Milhaud's *Saudades do Brazil,* is just such an example. (One suffers no discomfiture in analyzing a bitonal work as if it were syntonal because the bitonal work sounds like a performance of a syntonal keystream analysis with each keystream in a separate register, ignoring for the moment the relationship between the two keystreams.) [Example 5. Milhaud, *Saudades do Brazil/Corcovado,* mm.1-4]

The analysis reveals that, for the most part, the piece will be heard in the key of G major with some unresolved dissonances, the ninth on the downbeat of the first measure and the eleventh on the second beat of the second measure, for instance. It is not until the last beat of the excerpt with the occurrence of the C/C# simultaneity that the listener has

Example 5. Milhaud, Saudades do Brazil/Corcovado



any inkling that this is anything other than a tonal work. Even here, the clash is so quickly assimilated to the dominant of G that it sounds more like a spice than an essential harmonic element. Since C and C# are the only pitches not shared by the two scales, the composer will experience great difficulty producing a texture that does not lapse into tonality. It is obvious that the keys at the greatest remove around the circle of fifths, namely those that are tritonally or semitonally related, will offer the composer the greatest opportunity to create non-tonal sonorities. 2) Only the tritone/semitone relation gives the composer constant access to all twelve chromatic tones. This is a corollary to the first point. Figure 1 demonstrates the fusion of three pairs of scales **C/F**#, **C/Db** and **C/Ab**.[2]

Figure 1. Common tones between fused scales at the tritone, semitone and major third

```
C/F#:  C C# D D# E F F# G G# A A# B C
                   E#                B

C/Db:  C Db D Eb E F Gb G Ab A Bb B C
       C           F

C/Ab:  C Db D Eb E F    G Ab A Bb B C
       C           F    G
```

The first two fused scales form the chromatic collection. The two shared pc's are shown beneath each chromatic, F [E#] and B for **C/F#** and C and F for **C/Db**.

The third fused scale, **C/Ab**, is different. The F# is missing from the complete chromatic and each keystream shares three out of its seven pc's with the other. Thus, when related by major third, the keystreams are already beginning to show a lack of differentiation.

3) To introduce this lack of differentiation into the tritonal/semitonal superkey structure undermines its integrity and impedes the definition of the new surface.[3] The

major problem with Milhaud's account of bitonality is that by permitting bitonal relations at any interval, he makes it impossible for the listener or the analyst to define a coherent structure.[4]

One might reason that because the major and its relative minor share the same scale, the relative minor could substitute for its relative major as one of the constituent keys. So, for instance, one might suppose that since d# minor shares the same scale with F# major that the superkey **C/d#** would be equivalent to the superkey **C/F#**. If the C keystream stayed in C, then the other keystream would be free to modulate back forth between F# and d#. Figure 2 reveals why this does not work.

Figure 2. common tones between C major and d# minor

C/d#:	C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C
	B#		Cx			E#						B	

The top line of Figure 2 is the fused C major/d# minor scale. Underneath, the pc's held in common are singled out. Given the necessity of creating a dominant in the minor key, there are now at least three pc's held in common between the two scales, F [E#], B, and Cx. Voice leading may also invoke the melodic minor scale resulting in <u>four</u> shared pc's. Thus, substitution for the major by its relative minor breaks the tritone/semitone condition. The number of shared pc's between the keystreams equates **C/d#** with a key stream relation at the major or even the minor third. (Of course, there is nothing wrong with using the *natural* minor. However, as an analytical convention, all the church modes are assumed to be represented by the major scale into which they can be arranged. This is simply a convenience for the analyst that in no way implies that modes can be reduced to the major scale containing the same number of accidentals.)

The situation is even worse if both keystreams are minor, for instance **c/f#.** Figure 3 shows that, in this case, the keystreams share up to six pc's making them no more differentiated than two keystreams at a whole tone relation.

Figure 3. common tones between two tritonally-related minor scales

c/f#:	C	C#	D	Eb	E	F	F#	G	Ab	A	Bb	B	C
			D	D#		E#			G#	A		B	

Thus, if the tritone/semitone condition, which limits the number of shared pcs between the keystreams to two and only two, is to be maintained, the keystreams must be major.[5]

Admission of secondary dominants admits uncertainty and subjectivity into the analysis. Ultimately, the argument that secondary dominants are part of the key could be expanded on a Schenkerian scale to include any harmony, no matter how remote, within the realm of one tonic. One can imagine a scenario in which a keystream 1 in B major modulates to a long section in E major, its subdominant, while at the same time a keystream 2 in C modulates to a long section in E major, the V of its vi. The work would putatively project the semitonal superkey **B/C** though, in actual fact, during that section the keystreams in the same key would not sound syntonal at all.

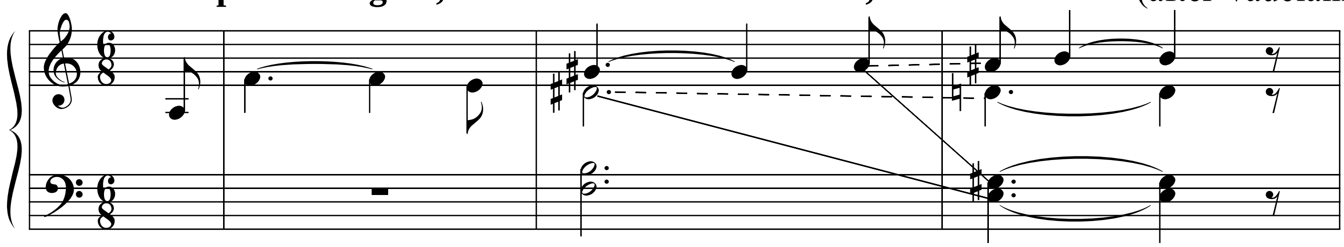
Syntonality justifies a totally chromatic texture with purely diatonic means. Therefore, altered tones and chromatic embellishments that chromaticize the keystreams, and that are then in further need of justification, are excluded. Part of the elegance of syntonality derives from its ability to account for every single note without recourse to events exterior to the system like 'altered tones' The keystreams must be in the tritone/semitone relation at every instant and within each key of the keystream the analysis must be completely diatonic. Although these proscriptions may seem overly rigid, they contribute strongly to the coherence of the system, thereby serving as the foundation to the enormous flexibility and freedom that, in fact, characterizes syntonal composition.

Consideration of dissonance invokes the voice leading aspect of keystream analysis. From the deep structural perspective, dissonance is a property of each keystream. It is defined and resolved precisely as it would be in a tonal piece of music. (Later, it will be defined in terms of the surface structure as difference in the interval string of a complete superchord.) In Example 4, the dotted lines connect dissonance to its resolution. The dissonance resolution in the first system is unexceptional. For instance, in the upper keystream, the seventh of the D dominant in m.107 resolves to the third of the G dominant in m.108 in the correct octave. Looking at the second system, in the lower keystream, the seventh of the Eb minor seventh chord in m.110 resolves to the B [Cb] of the G# [Ab] minor seventh chord in m.111. Here the resolution does not occur in the correct octave. Nevertheless, it is permitted because the dissonant note moves by conjunct motion. *A leading–tone or dissonance can resolve in another voice provided that it moves by step in its own voice to a note in either keystream.* This is an extension of the tonal practice illustrated by Example 6. [Example 6. Wagner, *Tristan and Isolde/Prelude,* mm. 1-3, (after Vauclain).[6]

The solid line connects the dissonance or leading-tone to its resolution. The dotted line connects the dissonance or leading-tone to the next note in its voice to which it moves by conjunct motion. It is fortunate that syntonality has this resource at its command, because without it, its ability to fuse the keystreams would be seriously impaired.

At this point it should be stressed that a keystream analysis is just that - an analysis. Although the keystreams should make musical sense, they are not the music.

Example 6. Wagner, Tristan and Isolde/Prelude, mm. 1-3 (after Vauclain)



One doesn't compose syntonal music by writing two beautiful pieces in keys that are a tritone apart and then combine them. Any good music is written directly to the surface, and syntonality is no exception.

At first, one might think that, given the potential access to all 12 pitch classes, most if not all twentieth century music would turn out to be syntonal. Upon reflection, however, it becomes clear why this would be extremely unlikely. As has been discussed earlier, syntonality is a special case of bitonality. The universality of syntonality as an analytical method would imply that all the masters of the twentieth century were bitonal composers, an improbable scenario.

A syntonal superchord is comprised of no more than eight pcs, the fusion of two seventh-chords drawn from keystreams at the tritone/semitone relation. Dissonance[7] resolution is conditioned by the following considerations.

1) Only 81 out of a possible 495 combinations of eight pcs are superchords. The other 414 contain some unessential[8] dissonance that must be resolved in the correct octave (or by step provided the resolution is taken by another voice).

2) Any simultaneities comprised of more than eight pcs of necessity contain some unessential dissonance that must resolve.

3) The essential dissonances (sevenths) within each superchord must also resolve.

Furthermore, even when no dissonance is present, the keystreams must be able to be extracted in a way that produces logical progressions.

The odds against the above factors being satisfied by chance are high enough that, even with only four or five voices, a prolonged syntonal texture is very unlikely unless the composer is creating it consciously. Therefore, it must be stressed, syntonal theory is designed for the analysis of syntonal works, not to explicate the unity in diversity of twentieth century chromatic music.

However, occasionally patches of syntonality have appeared intuitively. The first seventeen measures of the second movement to Bartok's *Piano Sonata* comprise one such instance that is demonstrated in Examples 7 and 8. Owing to the repetitive harmony of the opening measures, the keystream analysis will start on m. 6. Again, the solid lines refer to the note of resolution of a dissonance. When this occurs in another register, it must be accompanied by a dashed line that indicates the stepwise motion of the dissonance to a note in either keystream. [Example 7. Bartok, *Piano Sonata/II*, mm. 1-17] [Example 8. Bartok, *Piano Sonata/II*, mm. 1-17, keystream analysis]

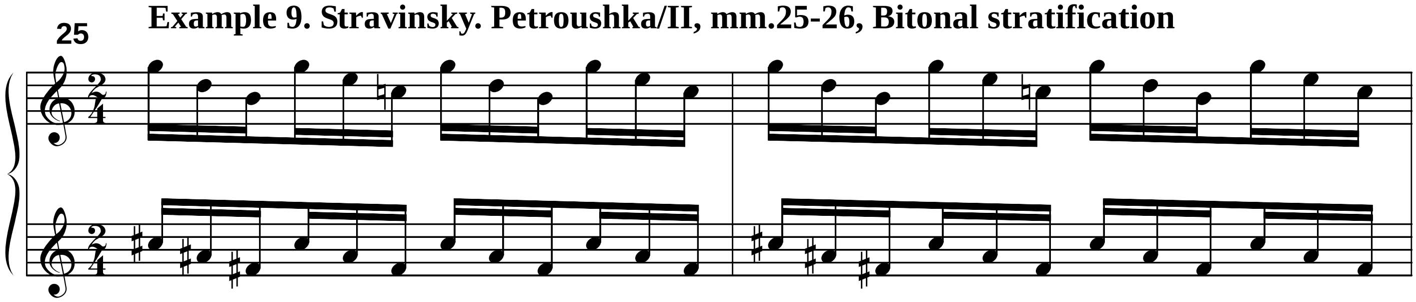
Before proceeding to a discussion of the superkey, it is instructive to observe bitonality turning into syntonality before ones ears, as it were, in a piece of real music. Example 9 excerpts mm. 25-34 from the second tableau of *Petroushka*. In mm.25-6, the constituent triads can be heard as independent units without too much trouble. Within a unitary sonic experience, the differentiated triads are perceptible because the keystreams are isolated in their separate registral spaces. (If the reader has difficulty perceiving this, play the excerpt again, this time with the F# triadal figuration an octave lower. The two keystreams will pop out with stark independence. Then play the excerpt as is. The keystream independence should still be perceptible.)

In mm.27-33, the F#/G→F#/C progression is completely fused. From m.25 to m.27 and ff., Stravinsky effected a transformation of bitonality into syntonality by means of a process of registral fusion. To recapitulate an earlier argument, if one hears the differentiated keystreams in mm.25-26, then one must assume that those keystreams are

Example 7. Bartok ,Piano Sonata/II, mm.1-17

Example 8. Bartok, Piano Sonata/II, mm.6-17, keystream analysis

Example 9. Stravinsky. Petroushka/II, mm.25-26, Bitonal stratification



Registral fusion

operative in mm.27-33 even though they have sunk below the level of conscious awareness.

Observe that in the act of notating the score, Stravinsky actually carried out a keystream analysis in mm.27-33 (RN50). He extracted the two keystreams and placed them on separate staves exactly as I did when I analyzed the excerpt from the *Hymn to Pan* Examples 3 and 4. The syntonal texture becomes more interesting in mm.29-30 because Stravinsky varies the simple alternation between keystreams with a pattern of 2+3. The left hand arpeggiates an F#+6 chord while the right hand arpeggiates a G^7 chord with a C appogiatura. The disposition of the two hands indicates that the separation of the music on two staves is not purely to facilitate performance. If that had been his purpose, he could have avoided the hand crossing at m.29 by alternating the first five notes between the hands.

In the examples from *Chez Petroushka* a registerally stratified excerpt is shown to be a perceptibly differentiated unity. Then a registrally fused version of the same harmonies is shown to be a perceptibly fused unity. But whether the total harmony is separated bitonality or fused syntonality, it is essentially the *same* harmony. The difference in sound is not due to any difference in harmony but to the vastly increased power of sonority in a totally chromatic world. As bitonality fuses into syntonality the integrity of the discrete elements dissolves as the integrity of the total entity is strengthened. Since the harmony remains the same, one can justify doing a keystream analysis on a completely fused syntonal work by the same means that one justifies doing an analysis of the constituent triads of a bitonally stratified work.

Surface Structure: The Superkey

The inversional square

The superkey is the fusion of two tonal keys a tritone or a semitone apart. There are six tritonal superkeys, **C/F#**, **Db/G**, **D/Ab**, **Eb/A**, **E/Bb**, **F/B**, and twelve semitonal superkeys, **C/Db**, **Db/D**, **D/Eb**, **Eb/E**, **E/F**, **F/F#**, **F#/G**, **G/Ab**, **Ab/A**, **A/Bb**, **Bb/B**, **B/C**. By convention, the superkey designation appears in bold or with a circle drawn around it. A letter-slash-letter not in bold refers to a superchord. The keystream whose tonic is lower on the chromatic scale C-B appears first in the superkey designation except in the case of the superkey **B/C**.

A tonal key is represented by seven diatonic seventh chords arrayed in a line, I, ii, iii, IV, V, vi, vii. The fusion of two tonal keys into a superkey is, in essence, a multiplication of the two keys which results in 7x7=49 superchords arrayed in a *square* seven by seven, I/I, I/ii, I/iii… ii/I, ii/ii, ii/iii etc. To present the many interesting patterns of order that result from this operation, it is convenient to array the superkey matrix as an **inversional square**. This is possible because each of the constituent tonal keys is capable of being arrayed in an **inversional line**. More importantly, the inversional square demonstrates that the inversional structural potential of tonality is assimilated in syntonality. As it turns out, it is only one of several structural potentials to be so assimilated, some others being a coherent organization of harmonic quality, and a systematic degree of proximity and remoteness between superkeys.

The knowledge that each major scale inverts around its second and flat sixth degrees is basic to contrapuntal technique in tonal music. Thus, one can say that C major, as well as F# major, invert at sum 4 because each pair of pcs in the inversional wedge sums to 4, mod 12. C inverts to F# at sum 10 and to Db at sum 5. [Example 10. Inversion in the keys of C, C to F# and C to Db]

Example 11 shows that the harmonic inversions of seventh-chords in the key of C major are balanced around vi, which inverts to itself. In the vi chord, the A inverts to G, C inverts to E, E inverts to C and G inverts to A. The brackets indicate that V inverts to vii, IV inverts to I, and iii inverts to ii. [Example 11. Seventh-chord inversions]

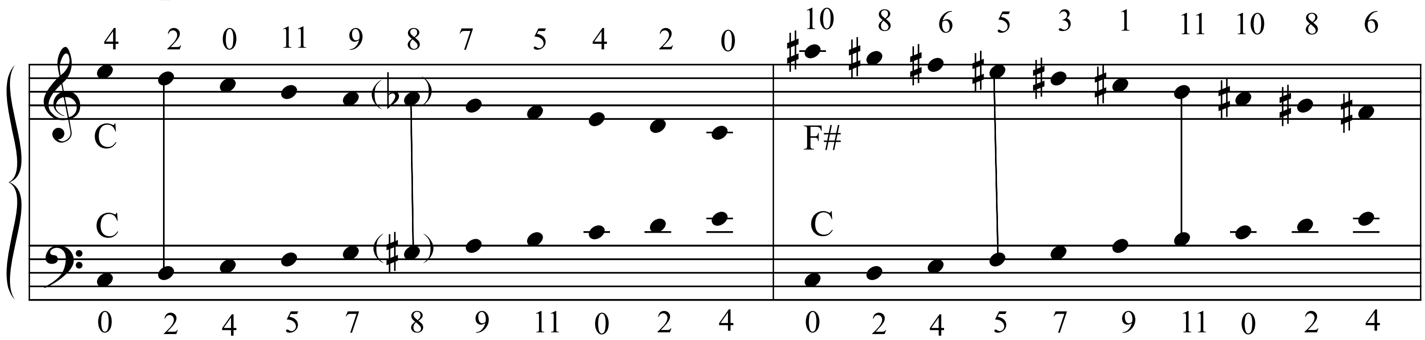
This is the linear inversional array that is multiplied in Figure 4 to produce the superkey array, the inversional square. [Figure 4. Superkey **C/F#** inversional square]

Each of the 49 positions in the matrix represents a syntonal superchord, which is a fusion of one seventh chord from C major with one seventh chord from F# major. The **position number** is indicated upper left in each cell. The matrix accounts for all the possible superchords within the superkey.

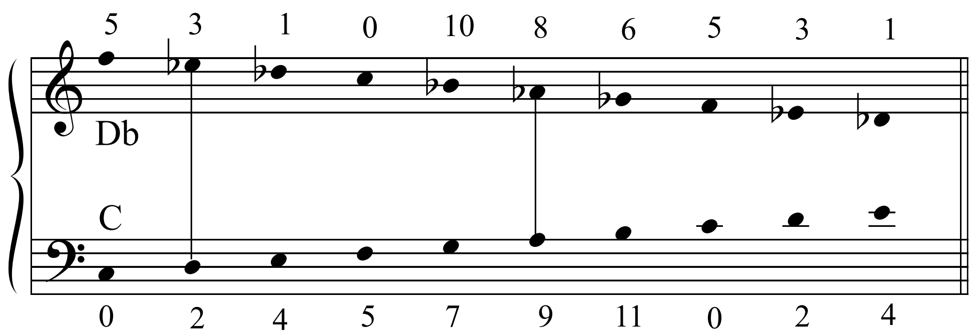
The **biletter** at each position describes the nature of the superchord in terms of its constituent seventh chords. A capital letter refers to a major seventh. A capital letter followed by a small x refers to a dominant seventh. A small letter refers to minor seventh and a small letter followed by a slashed circle refers to a half-diminished chord.

This matrix is an intermediate step that links the deep structure to the surface structure. The deep structure is still represented by the biletter designating each superchord. For instance, at position 1 the biletter d/a# informs one that this superchord derives from a fusion of the D minor seventh of C major and the A# minor seventh of F# major. Nevertheless, the matrix as a whole forms its own level of organization, one that could not be suspected from a consideration of keystream analysis alone. The biletter refers to the derivation of the superchord only, not to the experience of its conscious

Example 10. C scale inversion **C-F# inversion**



C-Db inversion



Example 11. Seventh-chord inversions

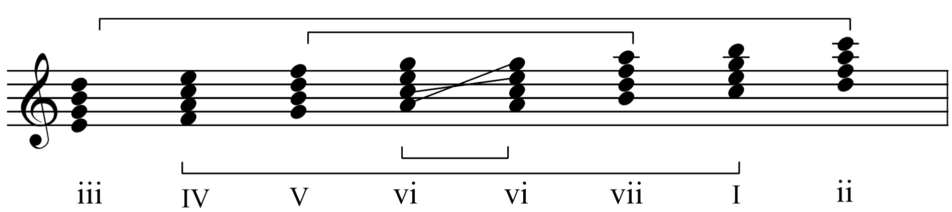


Figure 4. Inversional square, superkey C/F#

C/F#	iii	IV	V	vi	vii	I	ii
ii	1 d/a#	2 d/B	3 d/C#x	4 d/d#	5 d/e#ø	6 d/F#	7 d/g#
I	8 C/a#	9 C/B	10 C/C#x	11 C/d#	12 C/e#ø	13 C/F#	14 C/g#
vii	15 bø/a#	16 bø/B	17 bø/C#x	18 bø/d#	19 bø/e#ø	20 bø/F#	21 bø/g#
vi	22 a/a#	23 a/B	24 a/C#x	25 a/d#	26 a/e#ø	27 a/F#	28 a/g#
V	29 Gx/a#	30 Gx/B	31 Gx/C#x	32 Gx/d#	33 Gx/e#ø	34 Gx/F#	35 Gx/g#
IV	36 F/a#	37 F/B	38 F/C#x	39 F/d#	40 F/e#ø	41 F/F#	42 F/g#
iii	43 e/a#	44 e/B	45 e/C#x	46 e/d#	47 e/e#ø	48 e/F#	49 e/g#

perception. (Having made the intermediate step of the inversional square with its biletter designation, and after the investigation of its properties at this level, one will be poised to make an analysis of the superkey in terms of its purely interior relations.) The following discussion applies to all tritonal superkeys.

The inversional square for tritonal superkeys exhibits two types of inversional symmetry, horizontal and concentric perpendicular. The horizontal inversionality is manifested by the pattern-coded row pairs. Rows ii, I, and vii reading left to right invert to rows iii, IV and V reading right to left. Row vi inverts to itself, moving inward from positions 22 and 28 (or outward from the center of the chart, position 25, a/d#). The square can also be visualized as a coiled ribbon whose center is position 25. Position numbers that are equidistant from position 25 are inversionally related. (Since inversionally relayed pcs in **C/F#** sum to 4, this can be verified by assigning integers [C=0] to the pcs of any of its superchords and solving for the integers that will sum to 4 for each pc.)

The inversional symmetry of the concentric perpendiculars is indicated by the arrows at right angles. Looking at the outermost perpendicular, position 1 is self-inverting, 8 inverts to 2, 15 inverts to 3, 22 inverts to 4 and so on. Peeling the next layer of the onion, position 9 is self-inverting, 16 inverts to 10, 23 inverts to 11 etc. Given, this pattern, it is clear that each of the superchords running along the diagonal from positions 1 to 49 will be self-inverting.

In horizontal inversion each term of the biletter inverts within its keystream. In the earlier example ii inverted to iii within C major and iii inverted to ii within F# major. In the inversions along the concentric perpendiculars each term of the biletter inverts to the *other* keystream. For instance when position 8 inverts to position 2, the C major seventh of C major inverts to the B major seventh of F# major and the A# minor seventh of F# major inverts to the D minor seventh of C major. One observes that the keys of C major and F# major invert to each other at sum 10.[9]

The inversional square is a graphic demonstration of the organizational potential within tonality transformed into a higher order structure by syntonality. It illustrates the high degree of organization that the superkey imposes on the twelve pitch classes. However, it is important for the reader to keep in mind that the superkey is not necessarily tied to the inversional square. There are many other possible orderings of the superchords of a superkey in which its inversional structure would not be apparent. Thus, inversion is no more necessary to the syntonal surface because an inversional square exists than inversion is necessary to tonal composition because its chords can be ordered in an inversional line. It is simply a theoretical *potential*. As in tonality, a syntonal piece will not turn out to be inversional unless the composer makes it so.

Although it may seem that forty-nine superchords times eighteen superkeys should produce an inordinately large number of chords, it will be shown later that this is not the case. There is a significant degree of superchordal duplication between superkeys.

Syntonal quality

Just as in tonal music there are four diatonic seventh-chord qualities in a major key, major, minor, dominant and half-diminished, in syntonality there are eight superchordal qualities in a tritonal superkey, J[10], K, L, M, N, O (octatonic), MI and LI. Although the superkey squares the number of chords found in a tonal key, it only doubles the number of qualities.

The quality chart (two pages) for the tritonal superkey **C/F#** is displayed below. [Table 1. Quality Chart for **C/F#**]. The 49 superchords have been ordered in a way designed to maximize the grouping of like qualities in the same column while maintaining a numerical sequence among the **binumerals** along the rows. The binumeral is the Roman numeral equivalent of the biletter. For example, the binumeral I/IV, in the upper left hand corner of the cell in the first column and first row is equivalent to the biletter C/B in the superkey **C/F#**. Column 4, the central column of the chart has been set so that all the binumerals have like terms, I/I, V/V, ii/ii etc. Furthermore, they have been ordered down the column so that they conform to an upward circle of fifths pattern. Along each row the C major term (the left term) of each binumeral is held constant while the F# major term (the right term) varies. Reading from left to right, the right term decreases in rows 1, 2, 6 and 7 and increases in rows 3, 4 and 5. A glance at the chart will make all this clear.

The circled capital letters are the qualities. A small ‘s’ within the circle means that the superchord is a subset of the complete quality. A small ‘c’ next to the quality indicates that that superchord has a complete contingent of eight pitch classes. A negative number next to the quality indicates the number of pitch class duplications in the superchord. For instance, in the superchord in cell 1:1, the B root of the B major seventh is doubled by the B seventh of the C major seventh, so there is the indication –1 next to the J. A wedge appears above or below the pc that is duplicated. Since the keys of C and F# share only B and F [E#], only these pcs may be duplicated.

The number of duplications in each row is summed to the right of the chart. These sums form a symmetrical array around the central row 4. Row 4 has no duplications

Table 1. Quality Chart for C/F# (2 pages)

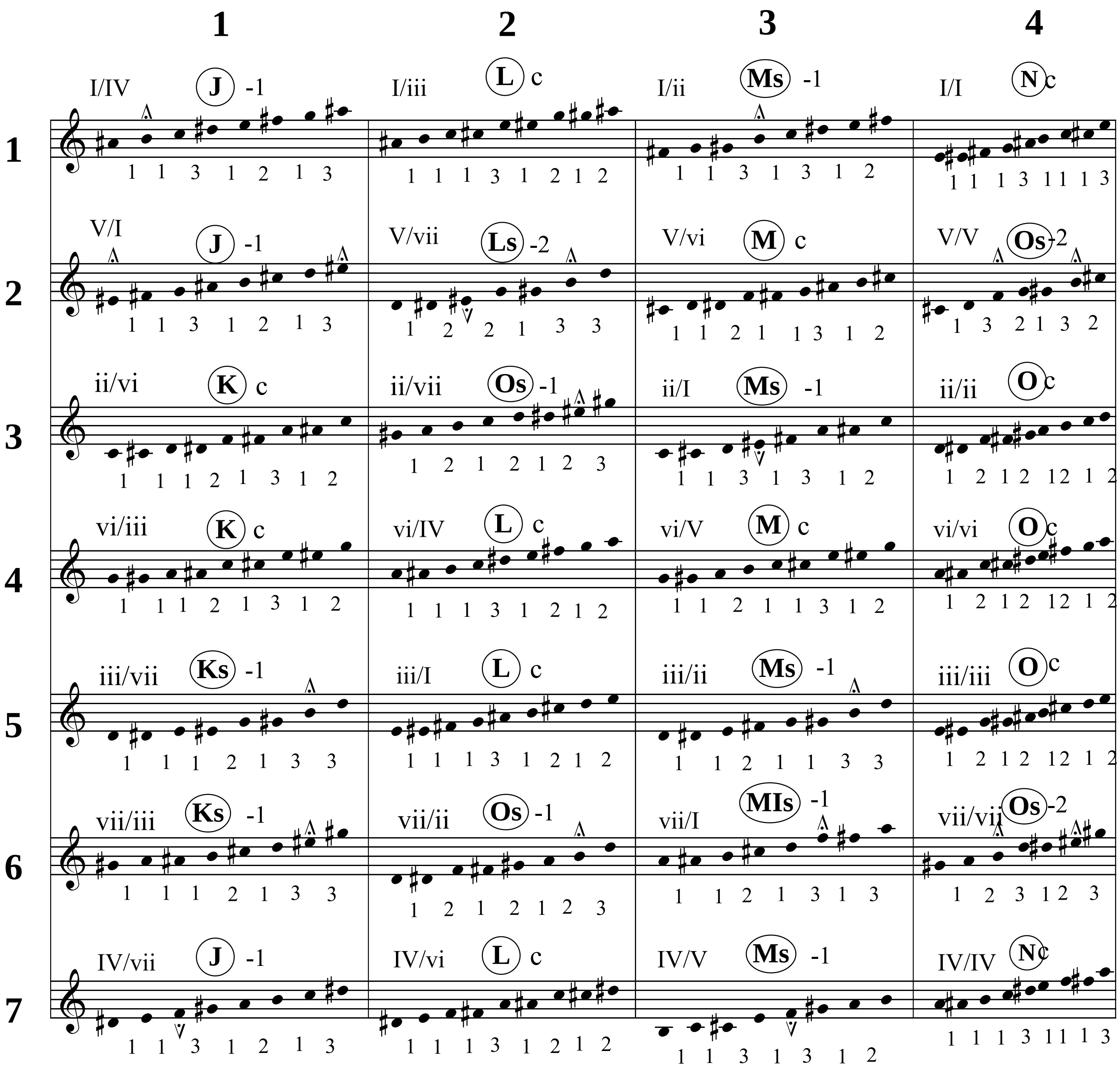
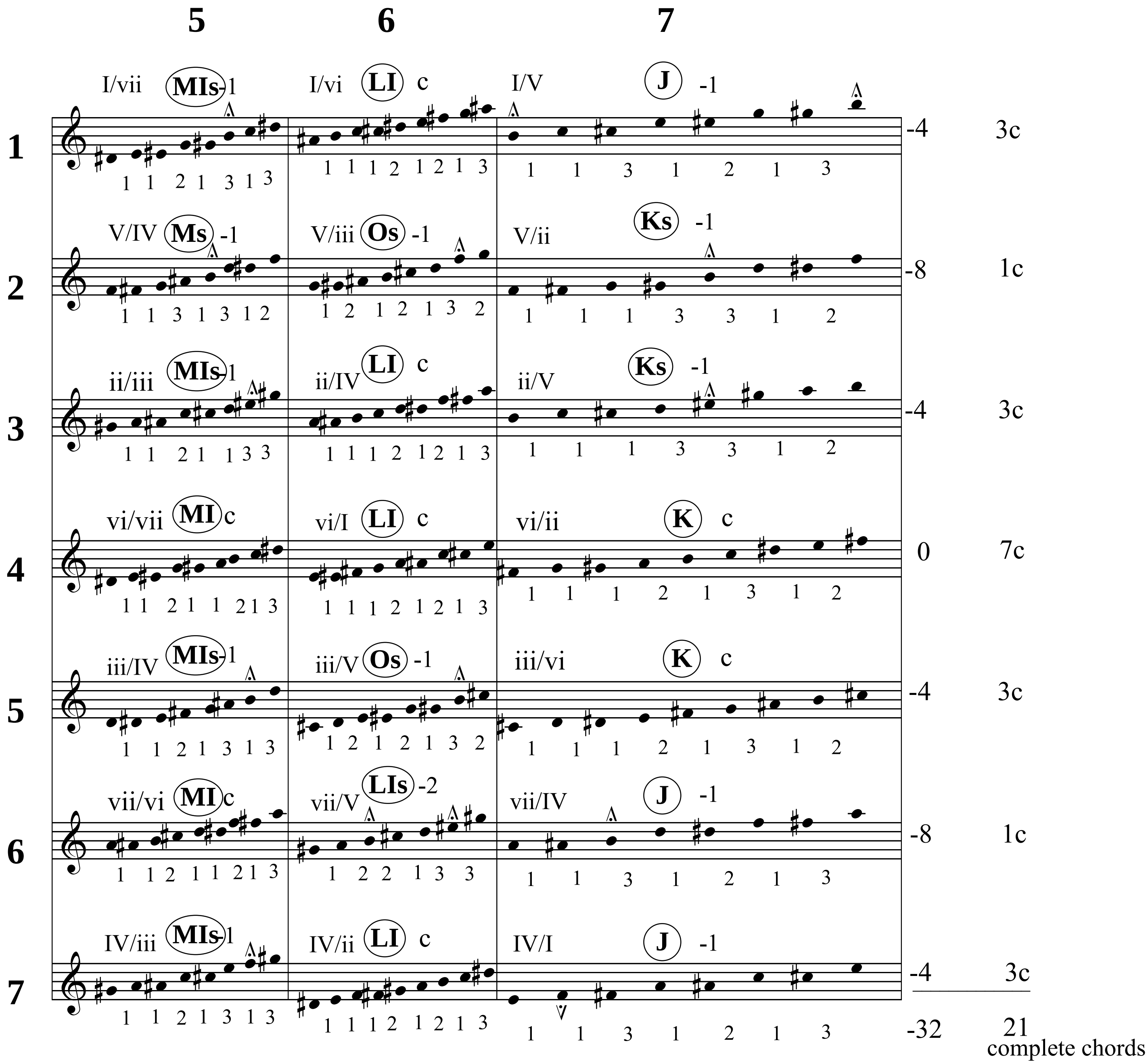


Table 1. Quality Chart for C/F# (2 pages)



because the vi^7 chord in C major, the left term of each binumeral in the row, contains neither of the pcs held in common between the keys of C and F#, F and B. To the right of the summed duplications are the numbers of complete superchords (those with eight pcs) in each row. These sums also form a symmetrical array. The sums at the bottom right hand corner of the chart show that there are a total of thirty-two duplicated pcs and twenty-one complete superchords in the chart. Later it will be shown that these figures also apply to the semitonal superkey quality chart.

The pcs of each superchord are represented by the stemless noteheads in each cell. The sequence of numbers between the noteheads in each cell is the **interval string** of each superchord. Each number refers to the number of semitones between the pitches that bracket it. So, in cell 1:1, there is one semitone between A# and B, one semitone between B and C, 3 semitones between C and D# etc. The total number of semitones must add up to 12 for the string to act like a continuous loop. Under this condition, parts of the interval string (henceforth known as string) can be taken from the end and added to the beginning or vice versa without changing the nature of the string. Thus the string of cell 1:1, 1131213, can also be written as 1312131 or 3121311 and so on. For the purpose of comparison it is convenient to write the string in a conventional order, which we will call the **normal order**[11]. To find the normal order, choose the string whose first term (leftmost) is the lowest. If these terms are the same, go to the second term and choose the string whose second term is the lowest. Proceed in this fashion until a term is reached that is lower than the corresponding term of any other version of the string. All the strings in Table 1 are in normal order.

Different orderings of the same string result in different inversions, all in closed position. This implies a difference in the applicability of normal order in tonality and syntonality. It would be silly to apply the above definition of normal order to triads because root position would be eschewed in favor of first inversion as the normal order! However, the definition of normal order presented above can be applied to syntonality because, as far as can be determined, at the level of surface structure *superchords have no roots*. In other words, there is no ordering or inversion of a superchord that is privileged above any other inversion.

In the quality chart for tritonal superkeys qualities J and K are confined to columns 1 and 7. L is confined to column 2. M is confined to column 3 with one exception, which migrates to column 5. N is confined to column 4. O appears in columns 4, 2 and 6. However, the appearances of Os in columns 2 and 6 could also be interpreted as subsets of L and LI respectively, increasing the elegance of the chart somewhat. I choose to label them O for phenomenological reasons; they *sound octatonic*. MI is confined to column 5 with one exception, which migrates to column 3, balancing the vagrant that migrated from column 3 to 5. LI is confined to column 6.

The quality chart has its own thoroughgoing inversional symmetry that reflects the inversional symmetry of the inversional square. (Compare Table 2 to Figure 4.) Column 1 reading top to bottom is the inversion of column 7 reading bottom to top. In the same way, column 2 inverts to column 6, and column 3 inverts to column 5[12]. The central column 4 inverts to itself, balanced around vi/vi, the central cell 4:4.

Figure 5 illustrates how the quality chart maps onto the inversional square. [Figure 5. Syntonal qualities mapped onto the inversional square]

Table 2. Inversional structure of the quality chart

	1	7	2	6	3	5	4
1	J	J	L	LI	Ms	MIs	N
2	J	Ks	Ls	Os	M	Ms	Os
3	K	Ks	Os	LI	Ms	MIs	O
4	K	K	L	LI	M	MI	O
5	Ks	K	L	Os	Ms	MIs	O
6	Ks	J	Os	LIs	MIs	MI	Os
7	J	J	L	LI	Ms	MIs	N

Figure 5. Syntonal qualities mapped onto the inversional square

C/F#	iii	IV	V	vi	vii	I	ii
ii	1 Mis	2 LI	3 Ks	4 K	5 Os	6 Ms	7 O
I	8 L	9 J	10 J	11 LI	12 Mis	13 N	14 Ms
vii	15 Ks	16 J	17 Lis	18 MI	19 Os	20 Mis	21 Os
vi	22 K	23 L	24 M	25 O	26 MI	27 LI	28 K
V	29 Os	30 Ms	31 Os	32 M	33 Ls	34 J	35 Ks
IV	36 MIs	37 N	38 Ms	39 L	40 J	41 J	42 LI
iii	43 O	44 Mis	45 Os	46 K	47 Ks	48 L	49 Ms

The superchords of column 4 of the quality chart are arrayed along the great diagonal running from lower left to upper ride, shaded with the diagonal stripe. All the M and MI superchords from columns 3 and 5 are arrayed along the diagonals above and below the great diagonal and in the upper left and lower right corners. These areas have no shading. The areas shaded light gray contain the L, LI and Os superchords from columns 2 and 6 of the quality chart. The areas shaded with the horizontal stripe contain the K and J superchords found in columns 1 and 7.

A glance at the figure discloses that all the qualities are symmetrically disposed about the great diagonal. The diagonals above and to the left of the great diagonal are the inversions of the diagonals to the right and below when the diagonals are read in opposite directions.

Finally, observe the symmetrical pattern of complete superchords (those containing eight pcs) mapped onto the inversional square in Figure 6. The shaded areas contain the complete superchords of the superkey. The unshaded area contains all the incomplete superchords, those that have either one or two pitch class duplications and therefore consist of only six or seven pcs. [Figure 6. Complete superchords of the superkey mapped on the inversional square]

Figure 7 is the inversional square for the semitonal superkey **C/Db**.

The inversional square for **C/Db** can be generated from the inversional square of **C/F#** by transposing the right terms of the 49 biletters up 7 semitones. Like the inversional square for the tritonal superkeys, the concentric perpendiculars invert but unlike the tritonal inversional square, the horizontal and verticals *do not*. The perpendiculars invert because each triad from either keystream inverts into the *opposing* keystream at sum 5. For

Figure. 6 Complete superchords of the superkey mapped on the inversional square

C/F#	iii	IV	V	vi	vii	I	ii
ii	1 Mis	2 Ll	3 Ks	4 K	5 Os	6 Ms	7 O
I	8 L	9 J	10 J	11 Ll	12 Mis	13 N	14 Ms
vii	15 Ks	16 J	17 Lis	18 Ml	19 Os	20 Mis	21 Os
vi	22 K	23 L	24 M	25 O	26 Ml	27 Ll	28 K
V	29 Os	30 Ms	31 Os	32 M	33 Ls	34 J	35 Ks
IV	36 MIs	37 N	38 Ms	39 L	40 J	41 J	42 Ll
iii	43 O	44 Mis	45 Os	46 K	47 Ks	48 L	49 Ms

Figure 7. The inversional square for the semitonal superkey **C/Db**.

C/Db

	iii	IV	V	vi	vii	I	ii
ii	1 d/f	2 d/G^{b}	3 d/A^{b}x	4 d/b^{b}	5 d/c∅	6 d/D^{b}	7 d/e^{b}
I	8 C/f	9 C/G^{b}	10 C/A^{b}x	11 C/b^{b}	12 C/c∅	13 C/D^{b}	14 C/e^{b}
vii	15 b∅/f	16 b∅/G^{b}	17 b∅/A^{b}x	18 b∅/b^{b}	19 b∅/c∅	20 b∅/D^{b}	21 b∅/e^{b}
vi	22 a/f	23 a/G^{b}	24 a/A^{b}x	25 a/b^{b}	26 a/c∅	27 a/D^{b}	28 a/e^{b}
V	29 Gx/f	30 Gx/G^{b}	31 Gx/A^{b}x	32 Gx/b^{b}	33 Gx/c∅	34 Gx/D^{b}	35 Gx/e^{b}
IV	36 F/f	37 F/G^{b}	38 F/A^{b}x	39 F/b^{b}	40 F/c∅	41 F/D^{b}	42 F/e^{b}
iii	43 e/f	44 e/G^{b}	45 e/A^{b}x	46 e/b^{b}	47 e/c∅	48 e/D^{b}	49 e/e^{b}

instance, in the case of positions 36 and 6, which invert along the outer perpendicular, IV/iii inverts to ii/I. (Recall that *within* a tonal key the following seventh chord inversions obtain: I inverts to IV, ii inverts to iii, V inverts to vii and vi inverts to itself.) The IV of C major inverts to the I of the opposing keystream, Db major, and the iii of Db major inverts to the ii of the opposing keystream, C major. Because C major inverts to Db major (and vice versa) at sum 5, the superchords as a whole also invert at sum 5.

However, as the reader will recall from chapter 7, the horizontal inversions were predicated on the condition that both terms of the superkey invert around the same axes and at the same sums. The semitonal superkeys do not meet this condition. The axes of inversion of their terms will always be a semitone apart. In this case, C major inverts around D and G# and Db major inverts around Eb and A.

Table 3 is the quality chart for the semitonal superkey, **C/Db**. [Table 3. The semitonal superkey quality chart] Like the tritonal superkey quality chart, it contains twenty-one complete superchords and thirty-two pitch class duplications. The superchords are also organized in a similar manner with the binumerals having like terms, beginning with iii/iii, proceeding up the circle of fifths as one reads down columns 1 and 7. Because in a semitonal superkey, binumerals with like terms produce K and J qualities rather than N or O, they are found in columns 1 and 7 rather than being concentrated in column 4 as they are in the tritonal quality chart. As a result of the different directionality of the rows (the right terms of the binumerals in rows 1, 2, 6 and 7 ascend reading left to right and those of rows 3, 4, and 5 descend), the binumerals with like terms appear in column 1 for rows 1, 2, 6, and 7 and in column 7 for rows 3, 4, and 5. This is not as

Table 3. Quality Chart for C/Db

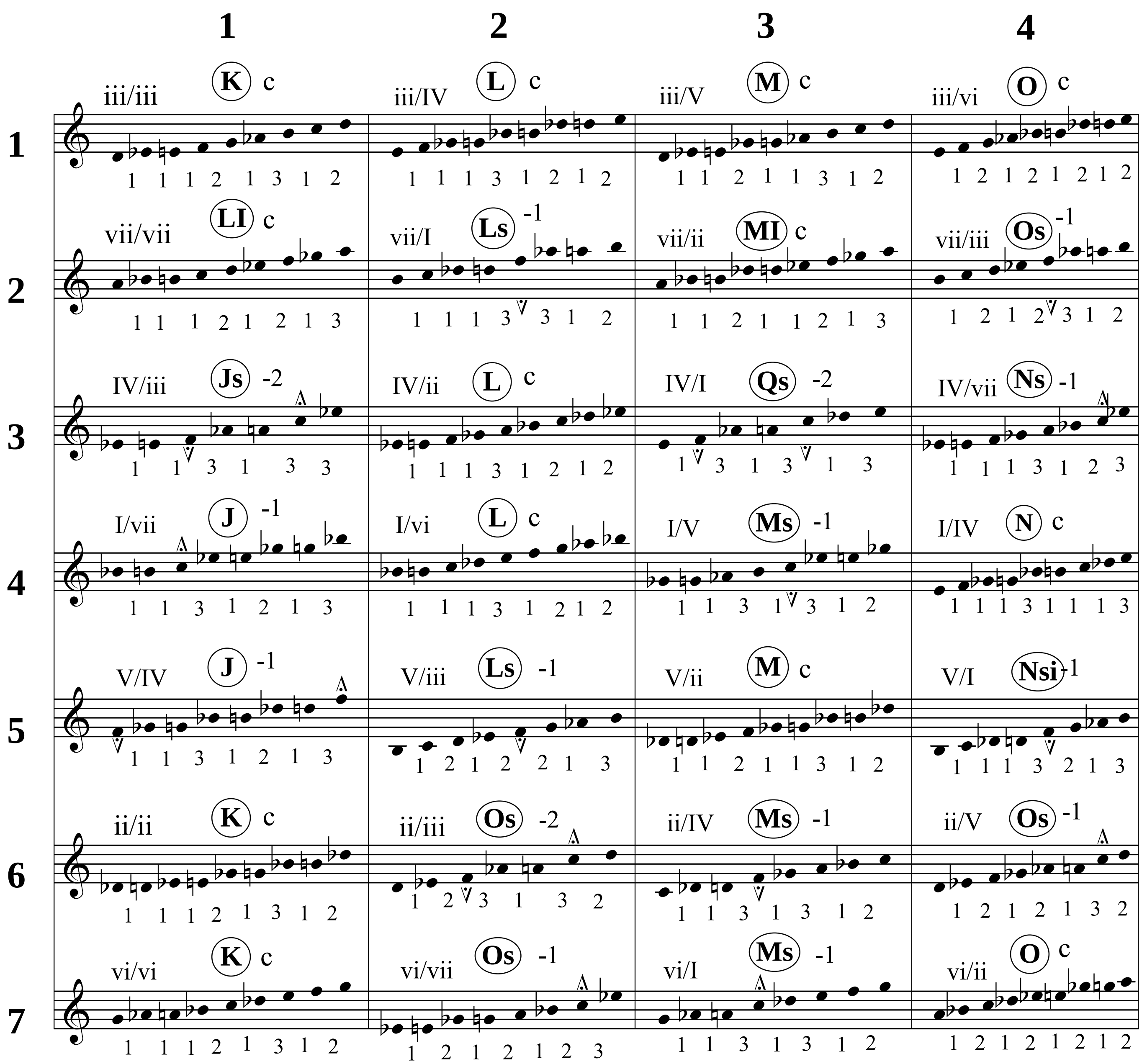
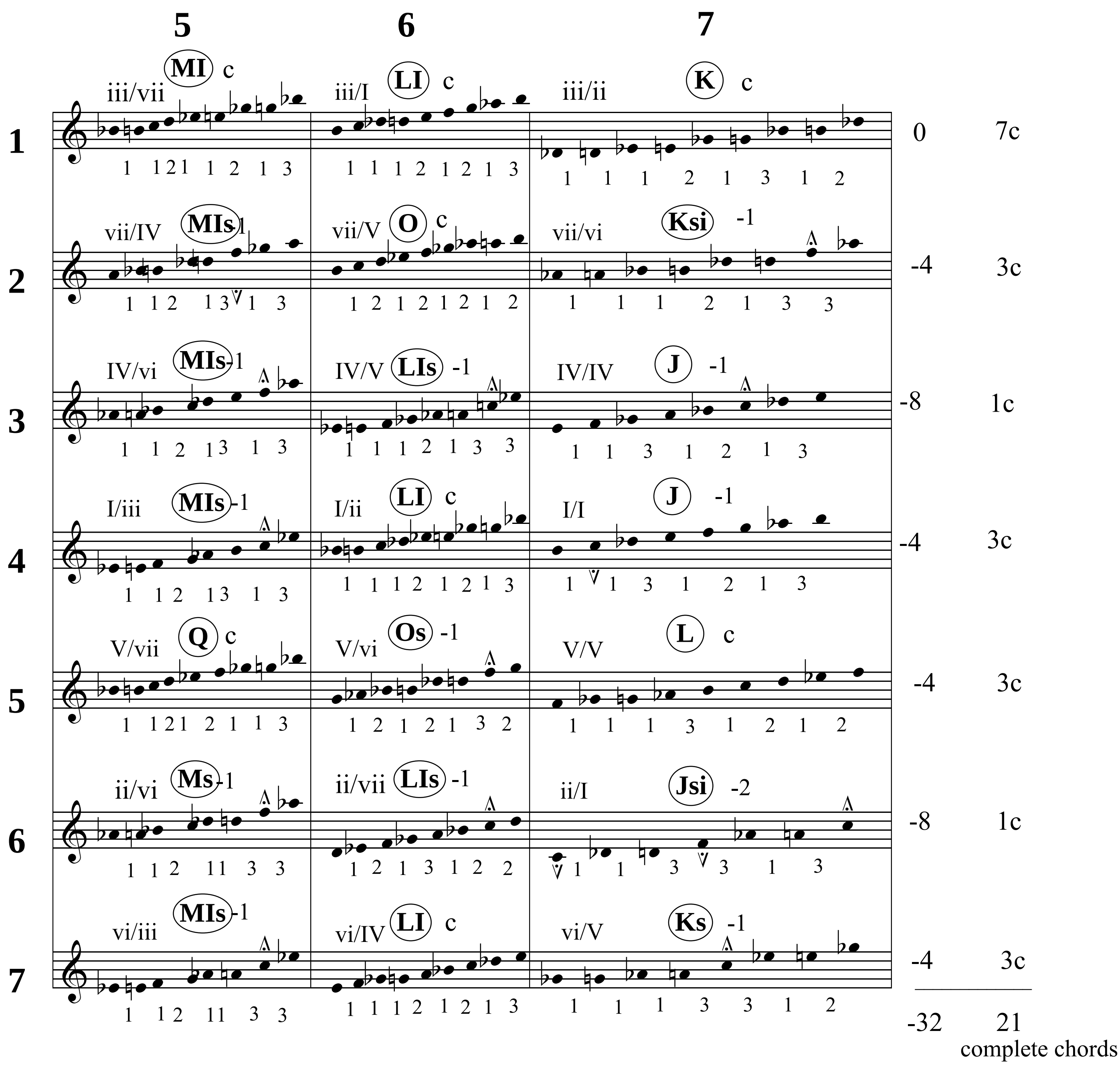


Table 3. Quality Chart for C/Db



complicated as it sounds. A glance at the chart should be enough to verify the pattern of superchord order.

As in the tritonal quality chart, K and J are confined to columns 1 and 7, M and MI are confined to columns 3 and 5 respectively with one vagrant interchange, N is confined to confined to column 4, and the forms of O not contained in column 4 appear in columns 2 and 6.

The differences between the tritonal and semitonal quality charts are as follows:

- Although rows 1 and 4 and column 4 are self-inverting, the quality chart for semitonal superkeys is not inversionally symmetrical as a whole. This reflects the difference between the inherent symmetricality of the tritonal superkeys whose constituent tonal keys share the same axes of inversion and the somewhat unbalanced semitonal superkeys whose constituent tonal keys do not.
- The semitonal superkeys possess a new quality Q, whose presence is registered in cell 5:5. Q is a very important quality because it is the only one that can single-handedly determine the identity of a superkey. On the other hand, there is the very interesting subset, Qs (cell 3:3), which is a 1-3 cycle. It can never assist in the identification of a superkey, because every one of its superchordal representatives can belong to any one of the 18 superkeys. Like its fellow chameleon, quality O, its syntonal function is analogous to the role played by the diminished seventh chord in tonal music.
- L and LI continue to be confined primarily to columns 2 and 6. However an LI and an L have migrated to columns 1 and 7 respectively.
- Some new subsets have been formed, Ns and Js for example.

- The Ms and MIs at the bottom of column 5 are now in the same column, not balancing each other in columns 3 and 5. This form of Ms or MIs is actually the same quality, both superchords having the same string, 1121133. However, since this subset form can be either a subset of M or MI, I have assigned one superchordal representative to each to maintain a balance between M and MI. As will be demonstrated shortly, quality subsets begin to lose their quality integrity. A full eight-note quality can be interpreted as one and only one quality. But a seven-note subset of that quality can be interpreted as a subset of two, and sometimes more, qualities. That is why, as the discussion proceeds, increasing emphasis will be placed on harmonies that can define complete (eight pc) superchords. (This does not mean that all eight pcs must be present. As long as the defining portion of the string is present, that suffices.)

Figure 8 maps the complete superchords in a semitonal superkey onto its inversional square, creating a flock-of-geese-in-flight pattern. [Figure 8. Complete superchords in a semitonal superkey] One can conceive of the inversional square as two cylinders interlocked at right angles. The Roman numerals at left are controlled by the cylinder on its side. As one rotates the cylinder, the roman numerals rotate as well. So, if one assigns the first rotation to the order of the Roman numerals in C major above, ii, I, vii, vi, V IV, iii, then the second rotation would be I, vii, vi, V, IV, iii, ii and so on.

As the cylinder on its side rotates to its second rotation, the erect cylinder that controls the roman numerals at top simultaneously rotates to its second rotation, which is IV, V, vi, vii, I, ii, iii. This preserves the essential nature of the square. The concentric perpendiculars continue to invert because the seventh chords of each keystream are still

Figure 8. Complete superchords in a semitonal superkey

C/Db

	iii	IV	V	vi	vii	I	ii
ii	1 Os	2 Ms	3 Os	4 Ms	5 Lis	6 Jsi	7 K
I	8 MIs	9 N	10 Ms	11 L	12 J	13 J	14 LI
vii	15 Os	16 MIs	17 O	18 Ksi	19 LI	20 Ls	21 MI
vi	22 MIs	23 LI	24 Ks	25 K	26 Os	27 Ms	28 O
V	29 Ls	30 J	31 L	32 Os	33 Q	34 Ns	35 M
IV	36 Js	37 J	38 LIs	39 MIs	40 Ns	41 Qs	42 L
iii	43 K	44 L	45 M	46 O	47 MI	48 L	49 K

Figure 9. Rotated pattern of complete superchords in the semitonal inversional square

C/Db

vii	I	ii	iii	IV	V	vi	
1 Ksi	2 L	3 Ms	4 O	5 MIs	6 Os	7 K	vi
8 LI	9 J	10 LIs	11 MI	12 Ns	13 Q	14 Os	vii
15 Ls	16 J	17 Jsi	18 LI	19 Qs	20 Ns	21 Ms	I
22 MI	23 LI	24 K	25 K	26 L	27 M	28 O	ii
29 Os	30 MIs	31 Os	32 K	33 Js	34 Ls	35 MIs	iii
36 MIs	37 N	38 Ms	39 L	40 J	41 J	42 LI	IV
43 O	44 Ms	45 Os	46 M	47 LIs	48 L	49 Ks	V

arrayed so they will invert into the opposing keystream, the I of C major, to the IV of Db, the vii of C to the V of Db etc. The seven rotations produce different pictures of the same square.

If one takes the fourth rotation, setting position number 1 to vi/vi, and then rotates the entire square 90° to the right in 2-dimensional space so that the Roman numerals of C are on top and those of Db are at *right*, one obtains Figure 9. Note that, although the contents of the complete superchords are slightly different, this *is the very same pattern which was obtained when the complete superchords were mapped onto the tritonal inversional square for the superkey* ***C/F#*** *in Figure 6.*

Although, as one would expect, the semitonal superkeys show less inversional symmetry than the tritonal superkeys, they share the following fundamental properties with them:

- The same number of superchords
- The same number of complete superchords
- The same number of total pitch duplications
- The same chord qualities, with the exception of Q which is unique to the semitonal superkeys
- The same pattern of complete superchords mapped onto the inversional square
- A very similar pattern of qualities mapped onto the inversional square

These shared attributes permit seamless movement between semitonal and tritonal superkeys.

In the most general terms, the inversional square provides a framework for thinking about the surface of syntonality. The notion of quality, which is defined by the interval string, will provide a means to discuss superchords without reference to their constituent keystreams.

Inter-superkey relations

Having dissected the anatomy of the superkey, the discussion proceeds to the relations between superkeys. A close relationship is expressed by a greater number of shared superchords Alternatively, the more superchords that are present in one superkey but not present in another superkey, the greater the distance between the two superkeys.[13] Such a superchord, which participates in differentiating one superkey from another, will be called a **difference chord**. A **difference chart** lists the superchords that differentiate a chosen superkey from a class of superkeys.

Table 4 (two pages) shows the difference chord relations between the superkey **C/F#** and the rest of the seventeen tritonal and semitonal superkeys, which are listed in the top row. Working inward from columns 1 and 18, they are symmetrically balanced around **C/F#**. Since **Eb/A** flanks **C/F#** symmetrically, it appears on both sides of the chart, in columns 6 and 13. The column at right lists all the possible difference chords. The 'x's indicate the superchords in **C/F#** not found in the superkey in whose column they appear. The bottom row sums the difference chords between **C/F#** and the superkey in whose column the sum appears.

Before discussing the attributes of the chart, it is important to know how to identify a difference chord. For example, cell 6:2 is empty. C/a# is not a difference chord between the superkeys **C/F#** and **Db/G** because the two superkeys share the superchord. Now take cell 1:2. Why isn't d/a# a difference chord between **C/F#** and **Db/G**? F# major and Db major share the a# minor seventh chord but C and G major do not share the d minor seventh. However, d/a# is not a complete superchord; the F [E#] is doubled. This

Table 4. Difference chart for C/F#   (2 pages)

	1	2	3	4	5	6	7	8	9	
	C/Db	**Db/G**	**Db/D**	**D/Ab**	**D/Eb**	**Eb/A**	**Eb/E**	**E/F**	**F/F#**	
1			x	x	x	x	x	x		d/a#
2		x	x	x	x	x	x	x		d/B
3					x	x	x			d/C#x
4		x	x	x	x	x	x	x		d/d#
5			x	x	x	x	x	x		d/F#
6			x	x	x	x	x	x		C/a#
7				x	x	x	x			C/B
8						x	x	x		C/C#x
9				x	x	x	x	x	x	C/d#
10						x	x	x		C/e#ө
11			x	x	x	x	x	x	x	C/F#
12				x	x	x	x			C/g#
13					x	x	x			bө/a#
14		x	x	x	x	x				bө/B
15					x	x	x			bө/C#x
16		x	x	x	x	x	x	x	x	bө/d#
17					x	x	x	x	x	bө/F#
18			x	x	x	x	x	x		a/a#
19	x	x	x	x	x	x	x			a/B
20	x	x	x	x	x	x	x	x		a/C#x
21	x	x	x	x	x	x	x	x		a/e#ө
22			x	x	x	x	x	x		a/F#
23	x	x	x	x	x	x	x			a/g#
24		x	x	x	x	x				Gx/B
25		x	x	x	x	x	x	x	x	Gx/d#
26						x				Gx/e#ө
27					x	x	x	x	x	Gx/F#
28						x				Gx/g#
29			x	x	x	x	x	x		F/a#
30	x	x	x	x	x	x	x	x		F/B
31	x	x	x	x	x	x	x			F/C#x
32		x	x	x	x	x	x	x		F/d#
33	x	x	x	x	x	x	x			F/e#ө
34			x	x	x	x	x	x		F/F#
35	x	x	x	x	x	x	x			F/g#
36				x	x	x	x			e/B
37				x	x	x	x	x	x	e/d#
38						x				e/e#ө
39				x	x	x	x	x	x	e/F#
40				x	x	x	x			e/g#
	8	15	23	30	35	40	35	23	8	

Table 4. Difference chart for C/F# (continued)

	10	11	12	13	14	15	16	17	18	
	F#/G	**G/Ab**	**Ab/A**	**A/Eb**	**A/Bb**	**Bb/E**	**Bb/B**	**B/F**	**B/C**	
1			x	x	x	x				d/a#
2	x	x	x	x	x	x				d/B
3				x						d/C#x
4	x	x	x	x	x	x				d/d#
5			x	x	x	x				d/F#
6			x	x	x	x	x	x	x	C/a#
7		x	x	x	x	x	x			C/B
8			x	x	x	x	x	x	x	C/C#x
9		x	x	x	x	x	x	x		C/d#
10			x	x	x	x	x	x	x	C/e#ɵ
11		x	x	x	x	x	x	x	x	C/F#
12		x	x	x	x	x	x			C/g#
13				x						bɵ/a#
14	x	x	x	x	x					bɵ/B
15				x						bɵ/C#x
16	x	x	x	x	x	x	x	x		bɵ/d#
17				x	x	x	x	x		bɵ/F#
18			x	x	x	x	x	x	x	a/a#
19		x	x	x	x	x	x			a/B
20		x	x	x	x	x	x	x	x	a/C#x
21		x	x	x	x	x	x	x	x	a/e#ɵ
22			x	x	x	x	x	x	x	a/F#
23		x	x	x	x	x	x			a/g#
24	x	x	x	x	x					Gx/B
25	x	x	x	x	x	x	x	x		Gx/d#
26			x	x	x					Gx/e#ɵ
27				x	x	x	x	x		Gx/F#
28			x	x	x					Gx/g#
29			x	x	x	x				F/a#
30	x	x	x	x	x	x	x			F/B
31		x	x	x						F/C#x
32	x	x	x	x	x	x				F/d#
33		x	x	x						F/e#ɵ
34			x	x	x	x				F/F#
35		x	x	x	x	x	x			F/g#
36		x	x	x	x	x	x			e/B
37		x	x	x	x	x	x	x		e/d#
38			x	x	x					e/e#ɵ
39		x	x	x	x	x	x	x		e/F#
40		x	x	x	x	x	x			e/g#
	8	23	35	40	35	30	23	15	8	

means that the d component is interpreted as a d minor seventh in C major and as a D dominant with a missing F# in G major, the E# [F] being taken by the a# component.

Another interesting example is the superchord bø/a# in cell 13:13. It cannot be used to differentiate **C/F#** from **F/B** because in **F/B** it is interpreted as Bb/c#. Bb/c# and bø/a# are **aliases**. Two superchords are aliases when their pcs are the same, but their constituent seventh chords are *different*.

In contrast, d/B is a difference chord between **C/F#** and **Db/G**. Whereas it is a superchord in **C/F#**, there is no way of partitioning its eight pcs to create two seventh chords which have as their roots one of the diatonic degrees of Db and G respectively.

Some notable attributes of the difference chart follow:

- There are 40 difference chords between **C/F#** and its most remotely related superkey, **Eb/A**. They are the 49 superchords of **C/F#** minus its 9 octatonic superchords (quality O). Each of the octatonic superchords can be interpreted in any one of the 18 superkeys. Thus, they cannot be difference chords.
- The order of the difference chords reading down the chart corresponds to their order in the inversional square (Figure 4) reading from position 1 to position 49. That means that, counting down from the top of the chart, difference chords are inversions of their counterparts, counting up an equal distance from the bottom of the chart.
- Superkeys which are symmetrically disposed around **C/F#** are differentiated from it by the same number of difference chords. For example, **Db/G** and **F/B** are both differentiated from it by 15 superchords- not the same 15, of course.

- The difference chords of the symmetrically disposed superkeys are inversionally related. For instance, the difference chords of **Db/G** reading down are the inversions of the difference chords of **F/B** reading up.
- The inversional relations in the difference chart are translated graphically into the spatial arrays of the xs. Columns 1 and 18, 2 and 18, and so on invert spatially. The pattern of xs is flipped around the axis of rows 20 and 21. (I.e. Column 1 reading down is column 18 reading up.)
- Each superkey has a unique difference chord profile, implying a unique relationship between **C/F#** and each of the other seventeen superkeys.
- To generate a superkey difference chart for one of the other tritonal superkeys, keep the x pattern and transpose the difference chords in the right column and the superkeys in the top row.
- One cannot use the sums at the bottom of the chart to determine the number of difference chords between any other two superkeys besides the difference chart superkey, in this case **C/F#**, and one of the seventeen superkeys listed at top. For instance, one cannot subtract 30 from 40 and say that there are 10 difference chords between **D/Ab** and **Eb/A**. There still will be 15 difference chords between them

Table 5, the semitonal difference chart shares the following attributes with the tritonal difference chart:

- Superkeys that are symmetrically disposed around **C/Db** are differentiated from it by the same number of difference chords.

Table 5. Difference chart for the semitonal superkey C/Db (2 pages)

	1	2	3	4	5	6	7	8	9	
	C/F#	**Db/D**	**D/Ab**	**D/Eb**	**Eb/A**	**Eb/E**	**E/F**	**F/F#**	**F/F#**	
1				x	x	x				bө/bb
2		x	x	x	x	x	x			a/bb
3		x	x	x	x	x	x		x	bө/cө
4	x	x	x	x	x	x	x	x	(x)	Gx/cө
5				x	x	x				bө/Db
6					x	x	x			a/Db
7	x				x	x	x	x	x	Gx/Db
8		x	x	x	x	x	x	x	x	bө/eb
9		x	x	x	x	x	x	x	x	Gx/eb
10		x	x	x	x	x	x		x	F/eb
11		x	x	x	x	x	x	x		e/eb
12					x	x	x			a/f
13	x				x	x	x	x	x	Gx/f
14	x				x	x	x	x	x	e/f
15				x	x	x	x	x		bө/Gb
16		x	x	x	x	x	x			a/Gb
17				x	x	x	x	x		Gx/Gb
18		x	x	x	x	x	x			F/Gb
19			x	x	x	x	x	x		e/Gb
20		x	x	x	x	x	x			d/Gb
21		x	x	x	x	x	x	x		C/Gb
22					x	x	x			C/f
23			x	x	x	x	x	x		C/eb
24					x	x	x			C/Db
25			x	x	x	x				C/cө
26		x	x	x	x	x	x			C/bb
27			x	x	x	x				C/Abx
28		x	x	x	x	x	x		x	d/eb
29		x	x	x	x	x	x		x	d/cө
30		x	x	x	x	x	x			d/bb
31			(x)	(x)	(x)	(x)	(x)	(x)		(e/eb)
32	x				x	x	x	x	x	e/Db
33	x		x	x	x	x	x	x	x	e/cө
34	x		x	x	x	x	x	x	x	e/Abx
35		x	x	x	x	x	x		x	F/cө
36		x	x	x	x	x	x			F/bb
37										F/Abx
38	(x)	(x)	(x)	(x)	(x)	(x)	(x)	(x)	(x)	(Gx/cө)
39	x				x	x	x	x	x	Gx/Abx
40		(x)	(x)	(x)	(x)	(x)	(x)			(a/bb)
41										a/Abx
	8	16	23	27	36	36	32	16	15	

Table 5. Difference chart for C/Db (continued)

	10	11	12	13	14	15	16	17	
	G/Ab	**Ab/A**	**A/Bb**	**Bb/E**	**Bb/B**	**B/F**	**B/C**	**Db/G**	
1									bɵ/bb
2		(x)	(x)	(x)	(x)	(x)	(x)		a/bb
3	x	x	x	x				x	bɵ/cɵ
4	(x)	(x)	(x)	(x)	(x)	(x)	(x)	(x)	Gx/cɵ
5									bɵ/Db
6		x	x	x	x	x	x		a/Db
7		x	x	x	x	x	x		Gx/Db
8	x	x	x	x	x	x		x	bɵ/eb
9	x	x	x	x	x	x		x	Gx/eb
10	x	x	x	x				x	F/eb
11	(x)	(x)	(x)	(x)	(x)	(x)			e/eb
12		x	x	x	x	x	x		a/f
13		x	x	x	x	x	x		Gx/f
14		x	x	x	x	x	x		e/f
15			x	x	x	x			bɵ/Gb
16		x	x	x	x	x	x		a/Gb
17			x	x	x	x			Gx/Gb
18		x	x	x					F/Gb
19	x	x	x	x	x	x			e/Gb
20		x	x	x					d/Gb
21	x	x	x	x	x	x	x		C/Gb
22		x	x	x	x	x	x		C/f
23	x	x	x	x	x	x			C/eb
24		x	x	x	x	x	x		C/Db
25	x	x	x	x	x				C/cɵ
26		x	x	x	x	x	x		C/bb
27	x	x	x	x	x				C/Abx
28	x	x	x	x				x	d/eb
29	x	x	x	x				x	d/cɵ
30		x	x	x					d/bb
31	x	x	x	x	x	x			(e/eb)
32		x	x	x	x	x	x		e/Db
33	x	x	x	x	x	x	x		e/cɵ
34	x	x	x	x	x	x	x		e/Abx
35	x	x	x	x				x	F/cɵ
36		x	x	x					F/bb
37			x	x	x				F/Abx
38	x	x	x	x	x	x	x	x	(Gx/cɵ)
39		x	x	x	x	x	x		Gx/Abx
40		x	x	x	x	x	x		(a/bb)
41			x	x	x				a/Abx
	16	32	36	36	27	23	16	8	

- The difference chords of the symmetrically disposed superkeys are inversionally related. For instance, the difference chords of **Db/D** (column 2) reading down are the inversions of the difference chords of **B/C** (columns 16) reading up.
- The inversional relations in the difference chart translate graphically into the spatial array of the xs. Like the tritonal chart, the pattern of x's inverts in both the horizontal and vertical dimensions. The pattern of xs is symmetrical around row 21 and column 9.
- The other semitonal difference charts can be generated following the procedure for the tritonal difference charts
- The sums at the bottom of the chart refer *only* to the number of difference chords between **C/Db** and the other seventeen superkeys. For instance, although **Eb/A** and **Eb/E** are both separated from **C/Db** by thirty-six difference chords; eight superchords separate them from each other.

The semitonal difference chart differs from its tritonal counterpart in the following ways.

- There are thirty-eight difference chords, not forty. Semitonal superkeys contain eight octatonic superchords, not nine as the tritonal superkeys. But they also contain a 1-3 cycle superchord (Qs) and two other six-pc superchords, IV/iii and ii/I, which cannot differentiate between any superkeys. Therefore, there are a maximum of 49-11=38 difference chords. Although there are thirty-eight difference chords, the greatest number that operates in any one instance is thirty-six.
- The difference chords of the tritonal difference chart are ordered according to the order of the horizontal inversions of its inversional square. Since there is no horizontal inversionality in the semitonal inversional square, the order of the difference chords in the semitonal difference chart are determined by the concentric perpendiculars of its inversional square.

- Note that the difference chords found on the corners of the perpendiculars of the semitonal inversional square invert to themselves. In order to preserve the inversional symmetry of the chart, those superchords appear twice, mirroring each other. Those superchords that are repeated are in parentheses. Those parenthesized x's should be omitted when calculating the sums of difference chords for each superkey that appear in the bottom row.
- One remarks, that only here are there 2 superkey pairs that share the same difference chord profile, **Eb/A-Eb/E** and **A/Bb-Bb/E**. However, as noted above, the fact that two superkeys share a difference profile does not mean there is no difference between them. Eight difference chords still separate the superkey pairs in question.

Not only are superchords ordered in elegant patterns within the superkey but also the superkeys are ordered elegantly within the syntonal universe. This order determines the likelihood that a particular superkey will follow another in a modulation. The closer the relation between superkeys, the more likely one will succeed the other. The most common succession of superkeys is that in which one keystream remains constant while the other modulates by fifth. Only eight difference chords characterize this relation.

The complete superchords

Having investigated inter-superkey relations to this point, one is in a position to identify the total number of unique superchords in the syntonal universe. Since any superchord of fewer than eight pcs can always be interpreted as a subset of a complete superchord of eight pcs, only complete superchords will be included in the tabulation. The superchord chart, Table 6 (2 pages), lists all the possible eight-pc superchords according to quality. There is no attempt to impose a superkey structure on the list. All superchords in the same column have the same string and therefore belong to the same

Table 6. THE COMPLETE SUPERCHORDS (2 pages)

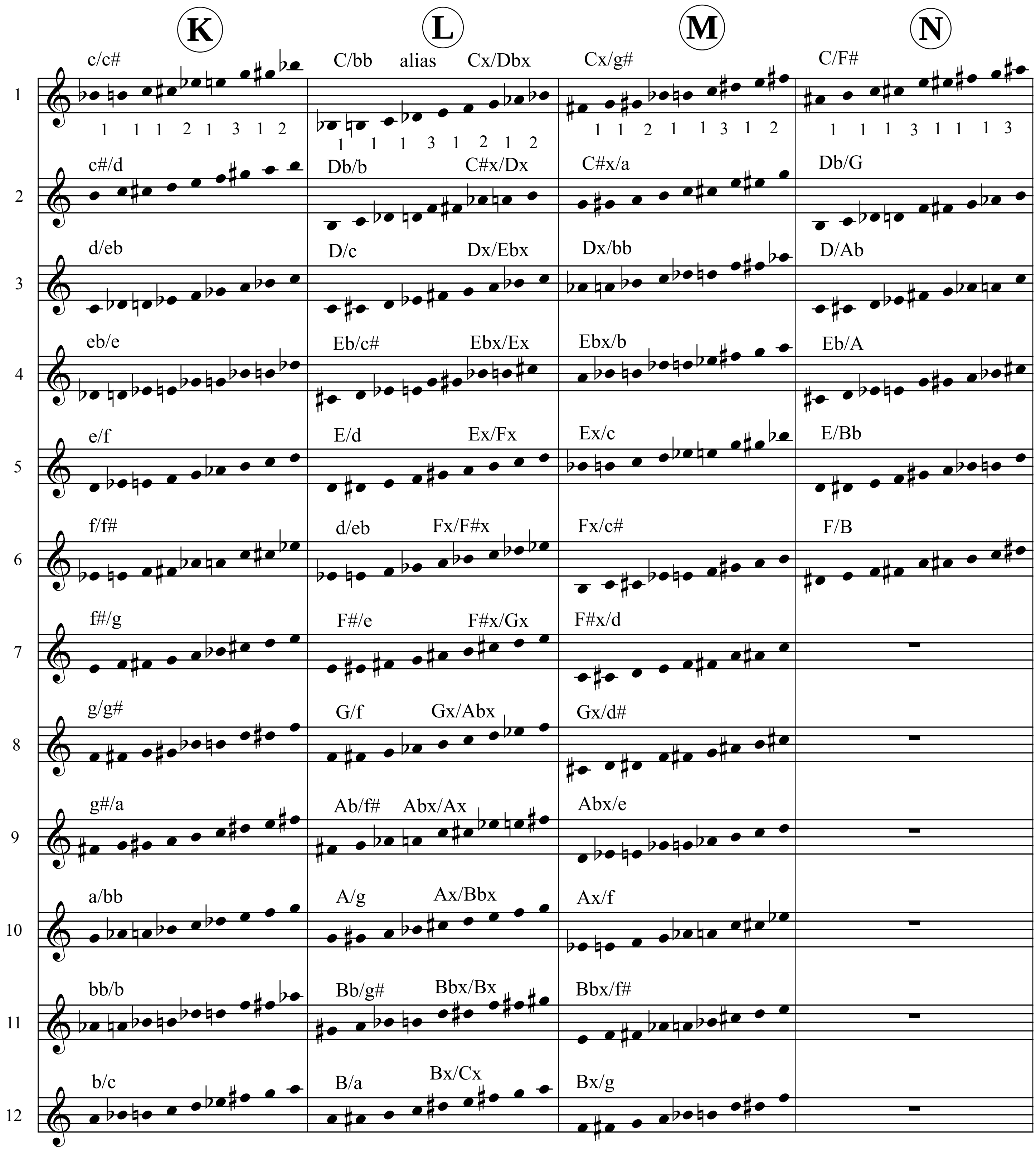
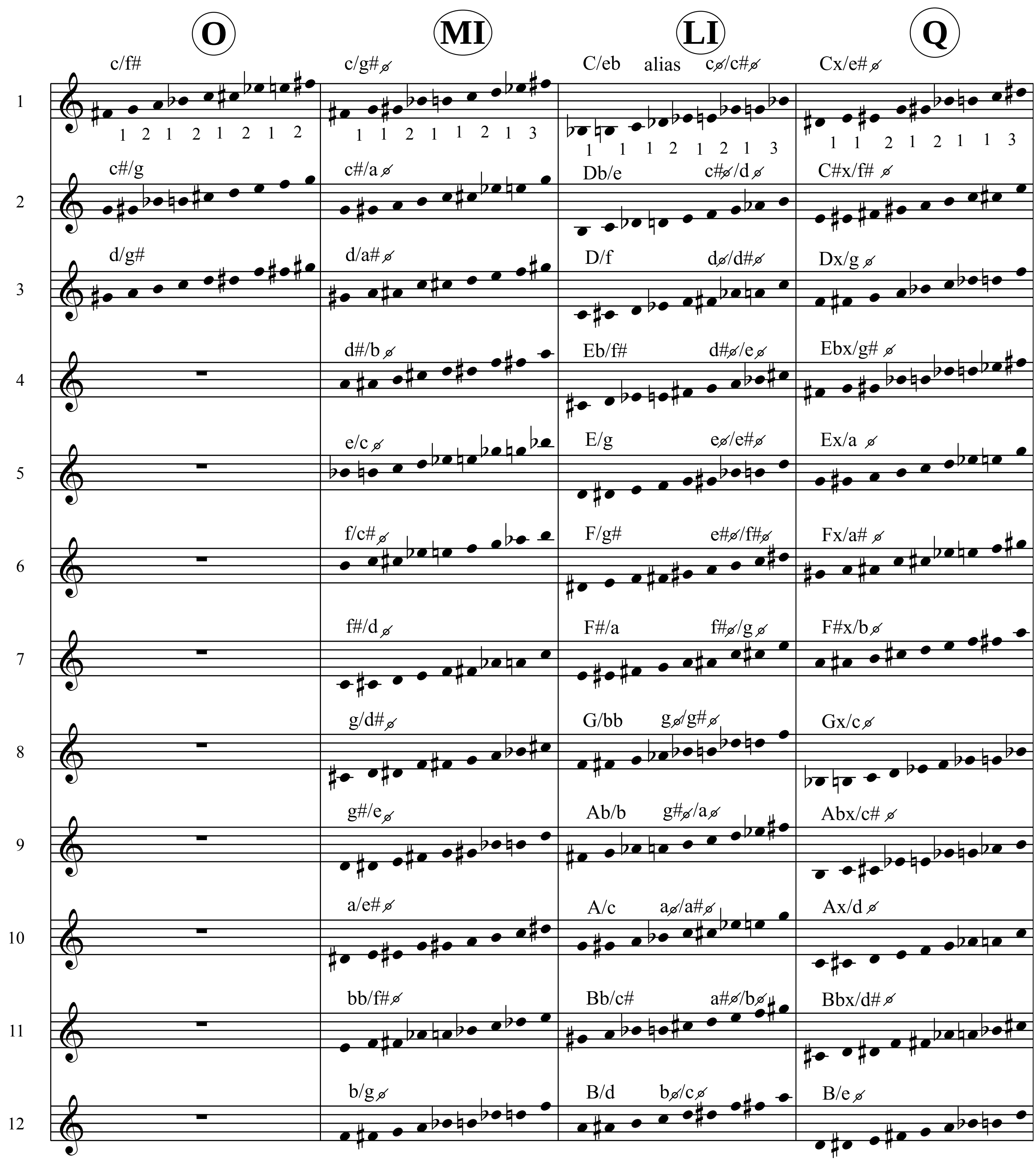


Table 6 continued



quality. The superchords are also identified by biletter. Those of the aliasing qualities, L and LI, are identified by two biletters. The operative biletter is determined by the musical context.

Every possible syntonal harmony is either one these 81 superchords or its subset. Each of the 81 complete superchords is not a harmonic class; it represents a specific harmony. There are only eight harmonic classes, the qualities K, L, M, N, O, MI, LI, and Q. Thus, like tonality, syntonality is able to account for a huge range of possible simultaneities with a relatively small number of basic chords. The difference is that the potential power of syntonality is much greater; the number of basic chords is comparable but the number of possible simultaneities is astronomically larger.

String analysis of syntonal harmony is a way station to the analysis of its pure surface.

- The interval string determines the quality of a superchord. It describes it phenomenologically as an integral unit without reference to the keystreams that form its deep structure, and therefore corresponds to the experience of the listener who will always interpret a superchord as a single, undivided sonic event.
- It can also help separate the keystreams when an intuitive keystream analysis flounders. A string analysis in combination with a separation of keystreams analysis provides the interface between deep and surface structure.

Because complete superchords are often not found in their complete eight-note state, it is desirable to find some subsets of the eight-note strings that will unequivocally define the quality. Table 7 lists the complete strings and the subsets that unequivocally define them. [Table7. Complete strings and their defining subsets]

Table 7. Quality-defining strings

K 11121312	111252	1213122	1211124	1112142
L 11131212	111423 111432	1113132	1131132	
M 11211312	114222	1122312	1131222	
N 11131113	111414	1113114	1114113	
O 12121212	121212	1212123	1212132	
MI 11211213	112224	1121322	1121124	
LI 11121213	111234 111324	1112124	1113213	
Q 11212113	113223	1131132	1123113	1121223

The qualities and their complete strings are displayed in the shaded column to the left. The subsets that unequivocally define the quality appear at right.[14] That means, for instance, that if there is a six note harmony with interval string 111252 it *can only* be a K superchord and no other. (However, it must be kept in mind that just because 111252 can only be a K harmony, it does not follow that it necessarily *is* a K harmony. It could be sandwiched between two simultaneities that make it impossible to interpret as that particular K harmony in a syntonal context.)

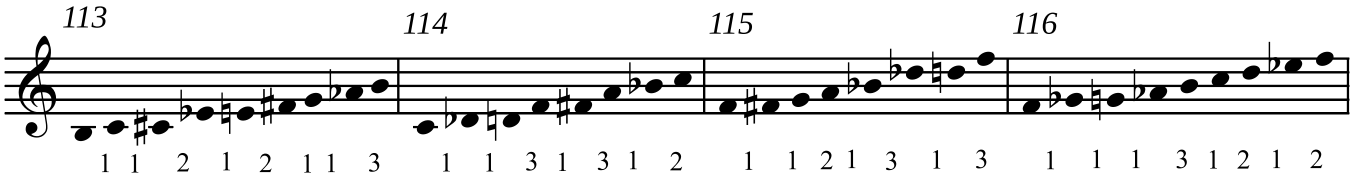
In order to demonstrate the possibility of ascertaining the relationship between syntonal surface and deep structures, four measures from *Hymn to Pan I* will be subjected to string analysis. [Example 12. Richardson, *Hymn to Pan/I*, mm. 113-116] [Example 13. The strings of the harmonies in Example12]

Clearly the harmonies of Example 12 change on the downbeat of each measure. Example 13 labels the string of each of the superchords. Looking at the quality chart, one determines that harmony 1 can only be quality Q. This is very convenient because it means the opening superkey is defined immediately. The string of Harmony 2 is not a defining subset but could represent the following qualities: K, M or Ms. Harmony 3 could be MI, MIs or Q. Harmony 4 can only be quality L. Table 8 shows that there is *only one* pathway through the maze of possibilities. [Table 8. Superchordal flow chart for *Hymn to Pan/I*, mm. 113-116]

The only possible pathway given the string succession defines one and only one superkey succession, **Db/D→Db/G→G/Ab→C/Db**. The schematic keystream analysis of Example 14 shows that there marriage between it and the string analysis. [Example 14. Keystream analysis, *Hymn to Pan/I*, mm. 113-116]

Example 12. Richardson, *Hymn to Pan I*, mm. 113-116

Example 13. ***Hymn to Pan*/I, mm.113-116, superchordal strings**



Go To The Next Page For Table 8

Example 14. Keystream analysis, ***Hymn to Pan*****, mm.113-116**

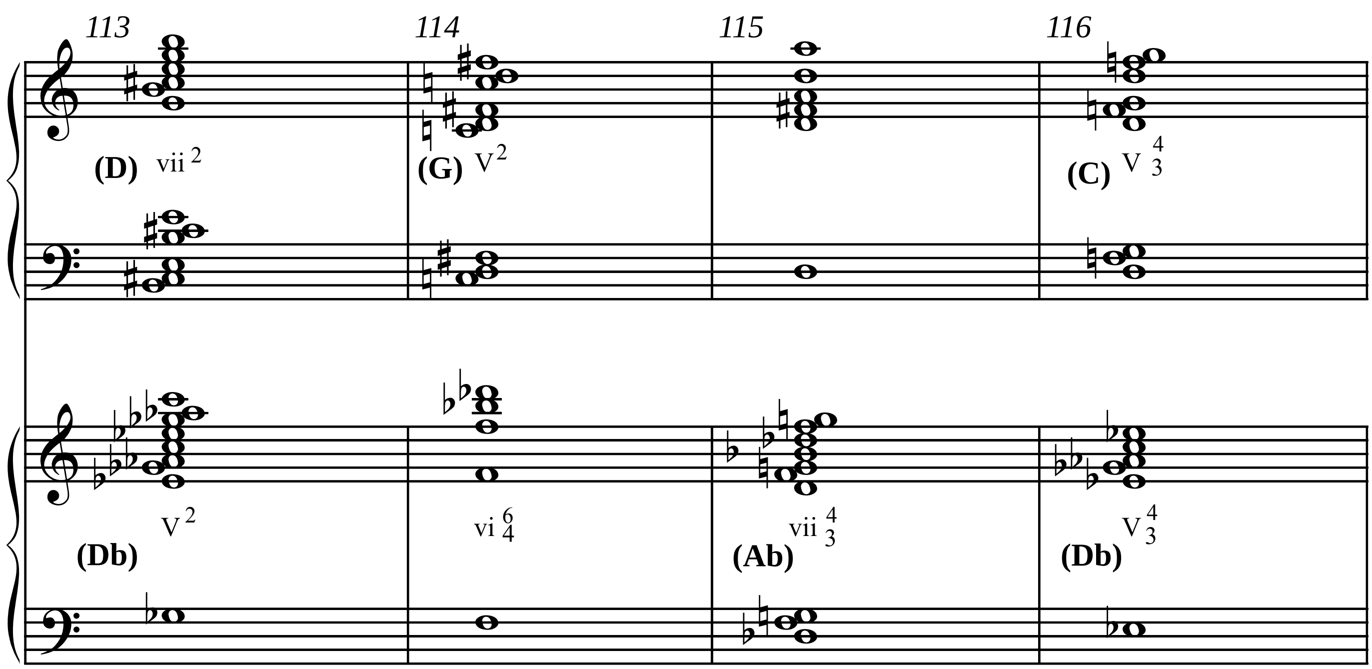


Table 8. Superchordal flow chart for *Hymn to Pan/I*, mm.113-16

Harmony 1	Harmony 2	Harmony 3	Harmony 4
	d/eb [K]	g/d#ɵ [MI]	
Abx/c#ɵ [Q] →	b♭/Dx [M] ↘	gɵ/D [MIs]	Gx/Abx [L]
	d/Gb [Ms]	Bb/eb [MIs]	G/f [L]
		gɵ/Dx [Q] ↗	

The preceding discussion has not been able to free the analyst from the deep structure but it has demonstrated an intimate and ordered relation between the deep structure (keystreams) and the surface structure (succession of superchordal strings). From here on, an effort will be made to cut free from the deep structure and describe all the interior and exterior relations of the superkey in terms of interval strings and their levels.

Toward the surface

In beginning the search for syntonal surface definition, the analysis will assume that:

1) All harmonies are complete eight pc superchords.

2) There are no non-harmonic tones.

If one can show that syntonal harmony can be defined in purely surface terms under these restricted conditions, then, in principle, it should be amenable to surface definition when the range of possibilities is expanded.

Because representatives of Q are non-aliasing superchords that are products of the fusion of a dominant seventh chord and a half-diminished seventh chord from their respective keystreams, they alone among superchords unilaterally define a superkey. For example, the Q superchord Gx/cø can only exist within the superkey **C/Db**. If one were to formalize the above one would sat that

Qa→**a+2/a+3**

where Q represents the quality and 'a' stands for the integer of the first pc of Q in normal order. So, if one looks at cell 8:Q in the superchord chart (Table 6), one sees that the string for the Q superchord Gx/cø begins on Bb or 10. Therefore

Q_{10}→**10+2/10+3** or Q_{10}→**0/1**

is just another way of saying Gx/cø→ C/Db but the difference in notation eliminates all trace of the keystreams. Furthermore, once one knows that Qa→a+2/a+3, one can define the rest of the complete superchords in the superkey in relation to each other. Table 9 shows the levels of the complete superchords in a semitonal superkey implied by Qa. [Table 9.The complete superchords in superkey a+2/a+3]

As an example, take the upper left cell, $K_{a+2.}$ When Q_{10}→0/1**,** then a=10 and K_{a+2} =K_0. One looks in the superchord chart for the K string that begins with pitch 0 (or C). That is the string for the superchord F/eb that is one of the superchords in the superkey 0/1 or C/Db.

If a Q superchord is not present, at least two superchords are necessary to define a superkey. Since the tritonal superkeys lack the Q quality, all tritonal superkeys require at least two superchords for definition. Table 10 lists the 22 combinations of two superchords that define a superkey, hereafter known as **definition pairs**. [Table 10. Superkey defining superchordal combinations]

An asterisk following the superkey designation indicates a tritonal superkey. The rest are semitonal superkeys. It is clearly more difficult to define a tritonal superkey than it is to define a semitonal superkey. An exclamation point following a definition pair means the superchords must appear in that order. Reversing the order would produce a tonally inconsistent progression. The other definition pairs can appear reversed.

Table 9. The complete superchords in superkey a+2/a+3

K_{a+2}	L_a	M_{a+3}	N_a	O_a	MI_{a+3}	LI_a
K_{a+3}	L_{a+5}	M_{a+4}	Q_a	O_{a+1}	MI_{a+11}	LI_{a+1}
K_{a+4}	L_{a+6}			O_{a+2}		LI_{a+6}
K_{a+9}	L_{a+7}					LI_{a+11}

Table 10. Superkey defining superchordal combinations

$K_a + K_{a+2\rightarrow}$	**a/a+1**	$L_a + M_{a+2\rightarrow}$!	**a+7/a+8**	$M_a + M_{a+1\rightarrow}$	**a+11/a**
$K_a + L_{a+5\rightarrow}$	**a/a+1**	$L_a + M_{a+11\rightarrow}$	**a+9/a+10**	$M_a + M_{a+6\rightarrow}$	**a+11/a+5***
$K_a + M_{a+11\rightarrow}$	**a+10/a+11**	$L_a + MI_{a+7\rightarrow}$	**a+9/a+10**	$M_a + MI_{a+2\rightarrow}$	**a+11/a+5***
$K_a + MI_{a+7\rightarrow}$	**a+10/a+11**	$L_a + MI_{a+10\rightarrow}$!	**a+7/a+8**	$M_a + MI_{a+7\rightarrow}$	**a+10/a+11**
$K_a + MI_{a+10\rightarrow}$	**a/a+1**			$M_a + MI_{a+9\rightarrow}$	**a+11/a**
$K_a + LI_{a+7\rightarrow}$	**a+10/a+11**			$M_a + LI_{a+1\rightarrow}$!	**a+4/a+5**
				$M_a + LI_{a+10\rightarrow}$	**a+11/a**
$N_a + N_{a+5\rightarrow}$	**a+2/a+8 ***		$MI_a + MI_{a+1\rightarrow}$	**a+3/a+4**	
			$MI_a + MI_{a+6\rightarrow}$	**a+3/a+9 ***	
			$MI_a + LI_{a+2\rightarrow}$	**a+4/a+10***	
			$MI_a + LI_{a+5\rightarrow}$!	**a+8/a+9**	

Conclusion

Syntonality assumes that the history of harmony is an unbroken continuity. The elements of tonality are not abruptly abandoned in favor of some completely new system of order (as in serialism) or a return to pre-tonal procedures (linear counterpoint, for instance). On the other hand, tonal principles are not applied anachronistically to a chromaticized context with which they are incompatible as happens in various applications of post-tonal theory. Rather, tonality sinks beneath the surface, as it were, to provide the deep structural foundation for a new surface that obeys its own principles of organization.

When two streams of diatonic tonality fuse to produce the syntonal surface, the listener loses awareness of the independent tonal streams. Nevertheless, their potential for harmonic articulation and continuity is taken directly into the syntonal surface. This is inherent in the process of syntonal formation.

<u>The very existence of syntonality depends on the condition that the constituent keystreams that constitute its deep structure are not accessible to the awareness of the listener and, by definition, can never be so.</u> If they could be independently distinguished, the syntonal surface would be disintegrated into bitonality. However, syntonality would also be impossible without the deep structural keystream organization discussed at length in this paper. To make a homespun analogy, a drop of rain would not feel like water unless it were H_2O (within a certain temperature range). But if one could feel the hydrogen and oxygen in it, it would not feel like water either.

How is the reader to verify the value of syntonality as both a compositional and a listening grammar? There is no other way other than a critical comparison of experience and theory. The defining attribute of syntonality is a consistent sense of harmonic articulation and continuity in a totally chromaticized context. If the listener hears these properties consistently in syntonal music, then justification of the theoretical details is almost beside the point. Alternatively, if the listener cannot detect any consistent form-building harmonic integration in the syntonal pieces he hears, he would be well advised to ignore the theory. Just about everything that a composer would have to know to write syntonal music is contained in the deep structural account of the keystreams. The remaining body of theory is an attempt to explain why anyone should want to carry out those procedures. However, any theory that limits itself to a deep structural explanation, no matter how cogent, cannot hope to thoroughly explicate the surface organization of conscious experience. Only a theory of the surface on its own terms can do that. Consequently, in the second part of this article I tried to develop a theory of syntonal surface structure which derives directly from the deep structure while it simultaneously creates its own patterns and levels of organization corresponding to the consciously experienced surface.

In the final analysis the theory will be judged by the works it attempts to describe, the extent to which the private acts of their composition intersect with the consciousness of society.

NOTES

[1] Vauclain, Constant. "An Experiment in Musical Texture." *The Musical Quarterly*, 25 (1965/2): 318-35. and "Bartok: Beyond Bi-modality." *The Music Review*. 41 (1981): 243-51. Vauclain coined the term "biscalarity" to identify the new system. On the suggestion of Maria Rose, in 1999 the system was rechristened "syntonality" because it was felt that what is being described is the fusion of two tonalities, not just the fusion of two scales.

[2] Superkeys are designated by the constituent keys in bold separated by a slash.

[3] For instance, in a hypothetical superkey **C/Ab**, one of the 49 superchords would be vii7/I^{7} or B D F A/ Ab C Eb G. If one reorders the pitches in the following way G Ab A B C D Eb F G, one determines that the superchord exhibits the interval string 11212122. This interval string does not exist for any superchord in a tritonal/semitonal superkey.

[4] There are two issues here, coherence and differentiation. A hypothetical syntonality based exclusively on major third relations would be coherent but undifferentiated relative to true syntonality.

[5] This is a point on which I differ with Vauclain. In June 1994 he wrote to me "As to the question of minor harmony in syntonality, my analyses of my own music (always after the fact, of course) not infrequently turn out to include it, and it appears to me to be merely the inversion of the major." He employs secondary dominants in his analyses as well. See Vauclain, "Bartok, Beyond", p.248, m.10.

[6] Vauclain, "Bartok, Beyond", p. 246.

[7] Recall that syntonal dissonance is simply the tonal dissonance *within* each keystream that must be resolved as it would in a tonal work. As will be explained, the dissonant simultaneities on the surface are defined by an interval string.

[8] The distinction between essential and unessential first made by Kirnberger still seems to me to be a valuable one. An essential dissonance is part of the harmony and resolves on the next harmony in the progression. An unessential dissonance is not analyzed as part of the harmony. It usually resolves over the same harmony. Sevenths can be essential dissonances, though not always. All other dissonances are unessential. Of course, this would not apply to the style of Debussy but that style is already taking its leave of the tonal world.

---

[9] By now the reader may be wondering why the superchords are constructed from constituent seventh chords and not some other tonal chord type, say triads or ninth chords. The reasons are as follows:

- Keystream analysis using seventh chords as the basic unit seems to best explicate my own syntonal practice as well as the patches of syntonality I have been able to discover in the standard literature. The triad as the basic unit of keystream analysis would involve the addition of something outside that unit on virtually every chord. (There is nothing *essentially* wrong with that approach. I actually worked out most of the superkey relations in this paper using the triad as the basic keystream unit before I decided that it did not adequately reflect compositional practice.)
- Along with the triad, seventh chords have long been considered basic building blocks of tonal harmony going back to Kirnberger and Marpurg. Marpurg accepted the dominant seventh as a fundamental chord of the first rank; Kirnberger accepted the other seventh chords as fundamental chords as well. (See Mekeel, Joyce. "The Harmonic Theories of Kirnberger and Marpurg." *Journal of Music Theory,* 4 (1960): 169-93. This idea has been enshrined in harmony textbooks ever since. There seems to be no reason to change it here. In practice, ninths and elevenths came to be treated as 'essential' only at the end of the nineteenth century when the tonal system was disintegrating.
- If one wants to create a structure, there has to be a point where some elements are accepted and others are excluded. This applies to *all* structures. In the case of the superkey, for example, if the constituent chords included the triad and its 7^{th}, 9^{th}, 11^{th} and 13^{th}, every superchord would consistent of all 12 pcs, vitiating any kind of harmonic structuring. In fact, in syntonality, there is nothing wrong with using 9ths, 11ths, and 13ths as long as they resolve, usually over the same harmony. However, to use them as part of the basic harmonic building blocks would make it impossible to formulate any theoretical harmonic description of the surface.

[10] Actually J is a pseudo-quality because, as will be shown later, it is a subset of L, M, MI or LI. In the semitonal superkeys there is a quality Q that does not appear in the tritonal superkeys. Defining a true quality as one that cannot be a subset of any other quality, there are still eight qualities K, L, M, N, O, MI, LI, and Q. Nevertheless, because J maintains its structural integrity and symmetrical placement throughout the tritonal quality chart, it seems reasonable to treat it as if it were a separate quality there.

---

[11] This is a term borrowed from set theory. It must be obvious, however, that the context in which it is used has nothing to do with set theory.

[12] Although K and O self- invert, the forms of Ks in columns 1 and 7 and Os in columns 2 and 7 *do not* self-invert. However, the form of Ks in column 1 does invert to the form of Ks in column 7 and the form of Os in column 2 does invert to the form of Os in column 6. (Later I distinguish the two forms of Ks as Ks and Ksi.)

[13] There is an obvious parallel to tonal theory in which the distance between two keys can be measured by the number of pcs their scales share. The difference is that tonal distance is measured in pcs whereas syntonal distance is measured in superchords. The reasons are clear. The distance between tonal keys can be measured triadically only as far as two moves around the circle of fifths. After that, there are no shared triads. On the other hand, superkey distance cannot be measured in scalar terms because all 18 superkeys share the chromatic scale.

[14] In determining which subsets of a particular quality unequivocally define that quality, it is not enough to exclude all of its subsets that are also subsets of some other quality. One must also exclude those subsets that are subsets of subsets of another quality. For instance the subset of L, 131212, is not a subset of any of the other complete qualities but it is a subset of the subset of K, Ks, 2121312. A complete string can be turned into a subset without losing its property of circular permutation if two of its terms are replaced by their sum. In this example, 11121312 was replaced by 2121312.

Captions

Example 1. Bitonal stratification

Example 2. Syntonal fusion

Example 3. Richardson, *Hymn to Pan/IV,* mm. 106-107

Example 4. Richardson, *Hymn to Pan/IV,* keystream analysis

Example 5. Darius Milhaud, *Saudades do Brazil/Corcovado*, mm. 1-4

Example 6. Richard Wagner, *Tristan and Isolde/Prelude*, mm. 1-3, (after Vauclain)

Example 7. Bela Bartok, *Piano Sonata/II*, mm. 1-17

Example 8. Bela Bartok, *Piano Sonata/II*, mm. 1-17, keystream analysis

Example 9. Igor Stravinsky, *Petroushka/II*, mm. 25-33, bitonal stratification and registeral fusion

Example 10. Diatonic scale inversions

Example 11. Seventh-chord inversions

Example 12. Dana Richardson, *Hymn to Pan/I,* mm.113-116

Example 13. Dana Richardson, *Hymn to Pan/I,* mm.113-116, superchordal strings

Example 14. Dana Richardson, *Hymn to Pan/I,* mm.113-116, keystream analysis

Figure 1. Common tones between fused scales at the tritone, semitone, and major third

Figure 2. Common tones between C major and d# minor

Figure 3. Common tones between two tritonally-related minor scales

Figure 4. Superkey **C/F#**, inversional square

Figure 5. Syntonal qualities mapped onto the inversional square

Figure 6. Complete superchords of the superkey mapped onto the inversional square

Captions in the order they appear in the body of the text

References

Mekeel, Joyce. "The Harmonic Theories of Kirnberger and Marpurg." *Journal of Music Theory,* 4 (1960): 169-193.

Vauclain, Constant. "An Experiment in Musical Texture." *The Musical Quarterly*, 25 (1965/2): 318-35.

_______"Bartok: Beyond Bi-modality." *The Music Review.* 41 (1981): 243-51.